Fall 2025 Math F113X

Exam 1

Name: _____ **Section:** ☐ **10:30 am (Leah Berman)**
☐ **11:45 am (Kevin Meek)**
☐ **online (Kevin Meek)**

Rules:

- Partial credit will be awarded, but you must show your work.
- You may have 1/2 of a standard page of paper ($8.5'' \times 5.5''$ or $11'' \times 4.25''$) of notes, both sides.
- Calculators are allowed.
- Place a box around your $\boxed{\text{FINAL ANSWER}}$ to each question where appropriate.
- Turn off anything that might go beep during the exam.

Good luck!

Problem	Possible	Score
1	20	
2	12	
3	20	
4	12	
5	18	
6	18	
Extra Credit	(6)	
Total	100	

1. (20 points)

A certain borough in Alaska has switched to using **Instant Runoff Voting (Ranked Choice Voting)** to determine the winner of its mayoral races.

In a recent municipal election, the preference schedule for the race was as follows:

	45	40	35	15	10	5
1st choice	Bork	Alton	Clay	Bork	Davis	Davis
2nd choice	Alton	Davis	Alton	Clay	Clay	Clay
3rd choice	Clay	Clay	Bork	Davis	Bork	Alton
4th choice	Davis	Bork	Davis	Alton	Alton	Bork

a. How many voters voted in the election? _____

b. How many voters are needed to have a majority of the votes? _____

c. Was there a winner after round 1 (that is, before anyone was eliminated)? Why or why not? Explain your answer.

d. Was anyone eliminated in round 1? Explain your answer.

e. Determine the winner of the election. Show your work clearly, in a way that someone else can follow. If you require multiple rounds, show the computations clearly, and clearly state which candidate is eliminated.

The winner of the election was _____ after _____ rounds.

2. (12 points)

Students in a dorm are voting on which Halloween activity the dorm should put on for local children. They are choosing among four choices, here conveniently labeled A, B, C, D. The preference schedule is below.

number of voters	45	15	15	10
1st choice	A	B	D	D
2nd choice	D	D	C	B
3rd choice	B	A	B	A
4th choice	C	C	A	C

a. How many voters participated? _____

b. Who was the plurality winner? _____

c. Find the winner under the Borda Count Method. (You must clearly show your calculations.)

d. Pick a fairness criterion that is violated in this election and provide a short explanation with supporting computation explaining why it is violated. Mark the box of the criterion that is violated.

- ☐ Condorcet Criterion
- ☐ Monotonicity Criterion
- ☐ Majority Criterion
- ☐ Independence of Irrelevant Alternatives (IIA) Criterion

e. Explain how you know this criterion is violated.

3. (20 points)

a. In the weighted voting system $[q : 10, 8, 2, 1]$, what is the **largest** value q can take? Justify your answer with a calculation.

b. In the weighted voting system $[q : 25, 7, 6, 3]$, find a value for q so that P_1 is a dictator. Justify your answer with a calculation.

c. Consider the weighted voting system [26:10,8,6,4,4,2,1].

i. Identify any players with veto power or state that none exist. Justify your answer.

ii. Explain why P_6, who has 2 votes, is not a dummy.

4. (12 points)

Consider the weighted voting system $[20 : 12, 10, 5, 3]$

a. Find all winning coalitions.

Winning Coalitions

b. Underline critical players in your winning coalitions above.

c. Fill in the following chart:

Player		Banzhaf Power Index
P_1		
P_2		
P_3		
P_4		

5. (18 points)

Suci and Matilda are dividing an ice cream cake with two flavors: hazelnut and chocolate. Each flavor takes up the same amount of cake, and the total value of the cake is $32.

a. What is a fair share of the cake? _____

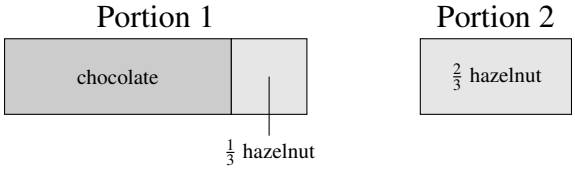
b. Suppose

- Suci likes hazelnut three times as much as chocolate.
- Matilda's valuation is listed in the table.

Fill in the table of values for Suci.

Person	Chocolate	Hazelnut	total value
Suci			
Matilda	28	4	

c. Suppose Suci is the divider. They divide the cake into two portions: one portion consists of all of the chocolate and 1/3 of the hazelnut, and the other portion consists of the remaining 2/3 of the hazelnut.



i. Why is that division a fair division for Suci? Provide supporting calculations.

ii. Which portion of the cake does Matilda choose? Why? Provide supporting calculations.

6. (18 points)

a. Four investors are dividing a piece of land valued at $320,000. One was chosen as the divider, and their values of the division (in thousands) are shown below. They decide to use the Lone Divider process to divide the land.

Person	Piece 1	Piece 2	Piece 3	Piece 4
Sonya	$90	$70	$ 80	$80
Cesar	$80	$80	$ 80	$80
Adrianna	$60	$70	$100	$90
Raquel	$70	$50	$90	$110

i. What is the value of a fair share to each investor? _____

ii. Who was the divider? Why? _____

iii. Circle the bids in the table that are fair shares to each investor.

iv. Is it possible to allocate the land so that everyone gets a fair share? If so, say who gets which piece of land; if not, say what happens next in the Lone Divider process.

b. Suppose three friends are dividing a cake that has been partitioned into three slices. They value the slices as follows:

	Slice 1	Slice 2	Slice 3
Paul	$2	$1	$6
Quentin	$3	$3	$3
Rosie	$1.50	$2.50	$5

i. Circle a fair share for each person.

ii. Explain why it's not possible to allocate one slice to each person at the first step.

iii. Explain what happens next in the Lone-Divider process.

Extra Credit (6 points)

Paula and John, and Willie are dividing up two items: a life-sized replica of Danny DeVito, and a live goat. They submit the following sealed bids for the three items.

	Danny Devito	Goat	total bid
Paula	\$130	\$230	
John	\$260	\$190	
Willie	\$310	\$170	

a. Determine each person's fair share (in dollars).

Paula's fair share	John's fair share	Willie's fair share

b. Determine which person gets each item.

Danny DeVito	Goat

c. Determine the **surplus**. Show your work!

d. What is the final allocation? Include items and how much each person pays and receives in the end? Show your work!

Paula:

John:

Willie: