Fall 2025 Math F113X

Exam 2

Name: _____ **Section:** ☐ **10:30 am (Leah Berman)**
☐ **11:45 am (Kevin Meek)**
☐ **online (Kevin Meek)**

Rules:

- Partial credit will be awarded, but you must show your work.
- You may have 1/2 of a standard page of paper ($8.5'' \times 5.5''$) of notes, both sides.
- Calculators are allowed.
- Place a box around your $\boxed{\text{FINAL ANSWER}}$ to each question where appropriate.
- Turn off anything that might go beep during the exam.

Good luck!

Problem	Possible	Score
1	10	
2	10	
3	12	
4	16	
5	14	
6	10	
7	16	
8	12	
Extra Credit	(6)	
Total	100	

1. (10 points)

On October 21, the cheapest available one-way, non-stop flights on Alaska Airlines between selected cities for flights on November 15, 2025, are listed in the following table. A – in the table means there is no available non-stop flight.

	Anchorage (ANC)	Seattle (SEA)	Honolulu (HNL)	Los Angeles (LAX)
Fairbanks (FAI)	\$109	\$179	–	–
Anchorage (ANC)		\$188	\$208	\$169
Seattle (SEA)			\$169	\$229
Honolulu (HNL)				\$229

Construct a weighted graph that represents the data. Make sure that your vertices are labeled in a useful way and that the edges and weights are clear.

2. (10 points)

Describe a **graph** that models a real world situation in which you would want to find each of the following. Explain what the **vertices**, **edges**, and **weights** represent, and what the circuit would tell you.

a. a minimum weight Hamiltonian circuit.

b. an Euler circuit.

3. (12 points)

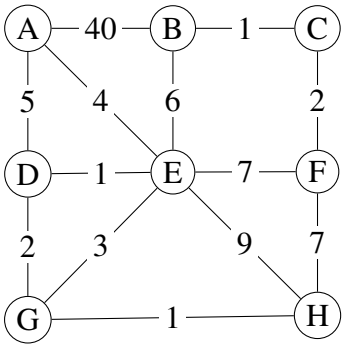
Recall that Kruskal's Algorithm says the following:

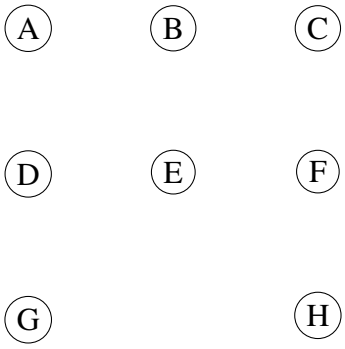
Kruskal's Algorithm: Select the cheapest edge in the graph that does not create a circuit. Stop when a spanning tree is obtained. Break ties by choosing the edge that comes earlier in the alphabet.

a. Use Kruskal's Algorithm to determine a **minimum cost spanning tree** in the following graph.

- Construct a minimum cost spanning tree in the second graph, with the **edges labeled with their weights**.
- Keep track of the steps of the algorithm, the edges that you are using, and the weights, in the table below.

For convenience, the edges of the graph are listed in sorted order by weight in the table below.





Sorted edges	weight	used?
BC	1	
DE	1	
GH	1	
CF	2	
DG	2	
EG	3	
AE	4	

Sorted edges	weight	used?
AD	5	
BE	6	
EF	7	
FH	7	
EH	9	
AB	40	

b. What is the total cost of the spanning tree you found? _____

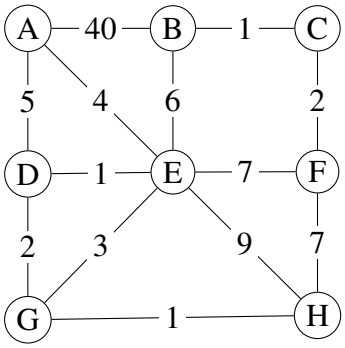
4. (16 points)

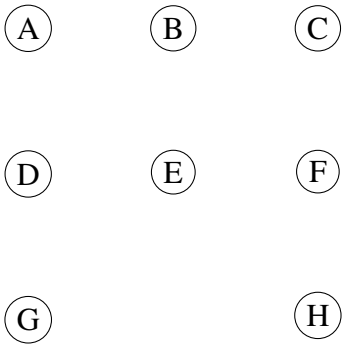
Sorted Edges / Cheapest Link Algorithm: Select the cheapest edge in the graph that does not create a vertex of degree 3 or close the circuit too soon. Break any ties by choosing the edge that comes earlier in the alphabet.

a. Use **Sorted Edges / Cheapest Link Algorithm** to find a **Hamiltonian circuit** in the following graph.

Keep track of the cycle, along with the weights, in the second graph.

For convenience, the edges of the graph, sorted by weight, are listed in order in the table below.





Sorted edges	weight	used?
BC	1	
DE	1	
GH	1	
CF	2	
DG	2	
EG	3	
AE	4	

Sorted edges	weight	used?
AD	5	
BE	6	
EF	7	
FH	7	
EH	9	
AB	40	

b. Write the Hamiltonian circuit you found, beginning with vertex *A*.

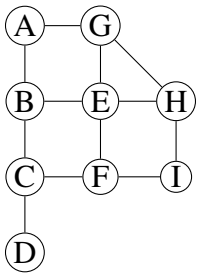
_____

c. What is the total weight of the Hamiltonian circuit? _____

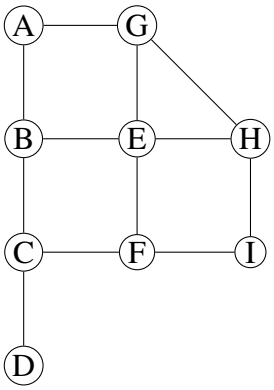
d. Is this the cheapest possible Hamiltonian circuit in this graph? _____ Explain your answer below.

5. (14 points)

a. Explain why the graph to the right does not contain an Euler circuit.



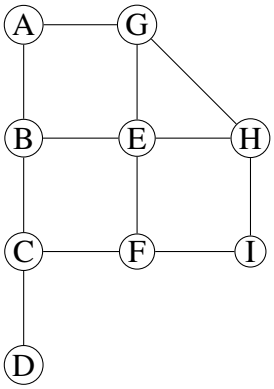
b. Eulerize the graph on the copy of the graph below **using as few edge duplications as possible.** Make your added edges very clear!



Added edges: _____

c. Find an **Euler circuit** in the eulerized graph by **drawing** the circuit on the graph **below this problem** and **listing** the vertices of the circuit in the blank below. (You will have to add your edges from part **b** in again!)

Offset your circuit slightly from the graph so that it is clear where the circuit is.



Euler circuit: _____

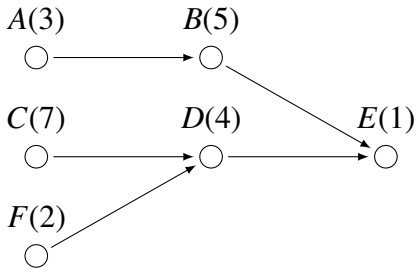
6. (10 points)

Construct a **scheduling digraph** corresponding to the following list of tasks and dependencies.

Task	Time	dependency
A	1 hours	
B	4 hours	
C	6 hour	
D	7 hours	A
E	2 hour	A,B
F	3 hours	D, E
G	5 hour	C,D,E

7. (14 points)

Consider the following digraph. The units for the tasks are **hours**.

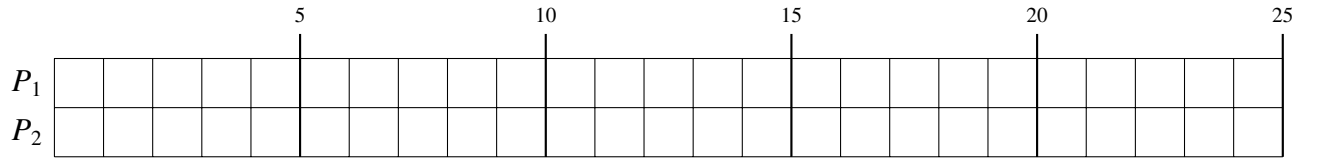


a. Construct a schedule for two processors using the **priority list**

$$D,\ A,\ E,\ F,\ B,\ C$$

Make sure to label the tasks in the schedule. You may use the table below to track your work.

time	
done	
ready	



b. How much **idle time** is in this schedule?_____

c. How long did this schedule take to complete? _____

d. Find the **critical path** _____ **critical time** _____

e. Is the schedule you found above optimal? Explain your answer: how do you know?

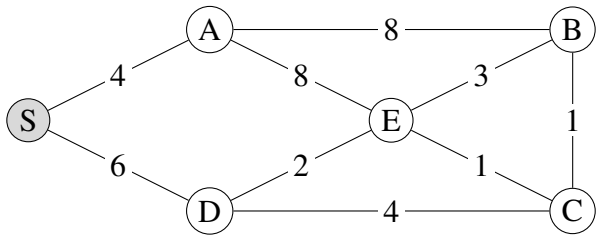
8. (12 points)

Recall Dijkstra's algorithm says the following:

Dijkstra's Algorithm

input: a graph with distances (weights) on the edges and a starting vertex, say s
output: the shortest distance between s and every vertex in the graph
rough strategy: All vertices get **tentative** distances to vertex s. One-by-one, vertices are explored and tentative distances are updated until minimum distances are obtained. Break ties alphabetically.

a. Use Dijkstra's algorithm to determine the distances between vertex S and each other vertex. Clearly show the steps of the algorithm in the space provided.



Explored?	vertices	tentative distances
	S	
	A	
	B	
	C	
	D	
	E	

List the vertices in the order you explored them.

_____

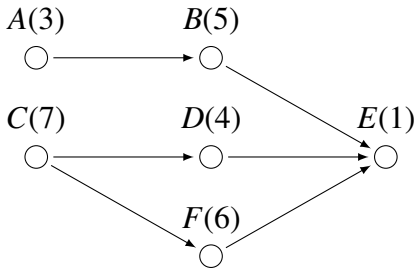
List the **final** distances to S.

vertex	S	A	B	C	D	E
distance from S						

b. Which vertex is farthest away from S? _____ How far is it? _____

Extra Credit: (6 points)

Consider the following digraph:



c. Construct the priority list for the digraph corresponding to the **decreasing time algorithm.**

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d. Label the vertices of the digraph according to the **backflow algorithm.**

e. Construct the priority list for the digraph corresponding to the **critical path algorithm**.
