202401 Math F113X - Numbers and Society/Midterm 1

James Gossell Semester: *Spring 2024*

Date Exam Taken: _____

Print Your Name Clearly	Proctor's Name

Start Time (to be filled out by Proctor)	End Time (to be filled out by Proctor)

Student Responsibilities:

- It is the student's responsibility to keep track of their time. Students are to complete the exam in one testing session.

- It is the student's responsibility to ensure all pages are included with the exam. The exam is **7** pages including this cover sheet.

Exam Specific Instructions:

- TIME LIMIT: **60 minutes**
- Outside materials that are allowed: **Calculators, writing utensils, and scratch paper are permitted.**

Math F113X, Spring 2024 Midterm 1

Name _____ Score _____

1. (20 points) Annie, Beth, and Carlos are running for class president. Their classmates fill out preference ballots and the results are given in the preference schedule below:

	6	**7**	**4**	**12**	**12**
1st choice	Annie	Annie	Beth	Beth	Carlos
2nd choice	Beth	Carlos	Annie	Carlos	Annie
3rd choice	Carlos	Beth	Carlos	Annie	Beth

a. How many total students voted?

b. How many votes are needed for a **majority**? Does any candidate have the majority of first place votes?

c. If the **plurality method** is used to determine the winner, who would win? *Explain.*

d. If **Instant Runoff Voting** is used to determine the winner, who would win? *Explain.*

e. If **Borda Count** is used to determine the winner, who would win? *Show your work!*

f. If **Copeland's Method** is used, would there be a clear winner? *Show your work!*

2. (15 points) A survey is conducted, and 15 panel members are asked to rank the four highest grossing movies of 2023: Barbie (B), Guardians of the Galaxy Part 3 (G), Oppenheimer (O), and the Super Mario Bros. Movie (S). The preference schedule is given here:

	5	**4**	**3**	**2**	**1**
1st choice	G	O	G	S	O
2nd choice	B	B	B	O	B
3rd choice	O	S	S	B	S
4th choice	S	G	O	G	G

a. Use **Borda Count** to determine the winner of the survey. *Show your work!*

b. With Borda Count, is it possible for the winner to receive no first place votes?

c. After the winner is announced, the producer of Guardians of the Galaxy is upset because his movie got a **majority** of the first place votes and still didn't win. Do you think he has a right to be upset? *Write a few sentences to explain your reasoning.*

3. (12 points) For each of the following problems, a **weighted voting system** is given. Determine whether there are any dictators and if anyone has veto power.

a. For the weighted voting system [16: 16, 11, 3, 1]

Are there any **dictators**? If so, who is a dictator?

Does any player have **veto power**? If so, who has veto power?

b. For the weighted voting system [51: 40, 30, 20, 10]

Are there any **dictators**? If so, who is a dictator?

Does any player have **veto power**? If so, who has veto power?

c. For the weighted voting system [30: 10, 9, 8, 7, 6]

Are there any **dictators**? If so, who is a dictator?

Does any player have **veto power**? If so, who has veto power?

4. (8 points) Ted is a 36-year-old single dad with 3 kids: Kelsey (age 12), Timmy (age 9), and Mandy (age 7). In order to make decisions for the household, Ted designs a weighted voting system based on age. In his system, each member of the family gets their age as their voting weight: Ted gets **36** votes, Kelsey gets **12** votes, Timmy gets **9** votes, Mandy gets **7** votes. It take a **majority** of the weight to pass any vote.

a. Write a **weighted voting system** to describe this situation.

b. Is anyone a **dictator** in this situation? If so, indicate who is the dictator.

c. Is anyone a **dummy** in this situation? If so, indicate all players who are dummies.

5. (15 points) In a small corporation, shareholders receive 1 vote for each share of stock they hold. There are 4 shareholders:

- Anderson owns **21** shares.
- Brown owns **20** shares.
- Clark owns **7** shares.
- Dunbee owns **5** shares.

Since there are a total of 53 shares owned, a majority (**27**) is needed to pass any business decision.

a. Write the following voting scenario as a **weighted voting system**.

b. List all **winning coalitions**.

c. In each of the winning coalitions above, indicate who is a **critical player**.

d. Calculate the **Banzhaf Power Index** for each player.

e. Based on the Banzhaf Power distribution, are there any **dummies** in this weighted voting system? *Explain.*

f. In your opinion, do you think this is a fair distribution of power? *Explain.*

6. (6 points) Recommend one of the fair division methods (divider-chooser, lone divider, or sealed bids) for each of the following scenarios. *Write a sentence to explain your reasoning.*

Scenario A: Two children are dividing a candy bar.

Scenario B: Bill and Ted are dividing their couch, tv, and game system that they purchased jointly back when they were roommates.

Scenario C: Lynn, Grace, and Cara are dividing time when they will each occupy their vacation cabin in Valdez.

7. (9 points) Adam, Bianca, Calvin, and Damian divide a plot of land using the **lone divider method**. Damian is selected to be the divider and divides the land into four sections, equal in his eyes. The table below represents the value of each section in each person's eyes.

Person	Section 1	Section 2	Section 3	Section 4
Adam	10%	30%	15%	45%
Bianca	10%	50%	30%	10%
Calvin	20%	20%	20%	40%
Damian	25%	25%	25%	25%

a. In the table above, **circle** each percentage that represents a **fair share**.

b. With this division, is it possible for each person to get a fair share?

c. If it is possible, explain who will get each section. If it is not possible, explain which section Damian should take and what happens next.

8. (15 points) The dwarves Thorin, Balin, Fili, and Kili are dividing up some loot that they found in a dragon's lair. The loot consists of a mithril shirt, a gold cup, and The Arkenstone. They decide to divide the loot using the **sealed bids method**. The table below shows how many gold coins each dwarf bid on each item.

	Mithril Shirt	**Gold Cup**	**The Arkenstone**
Thorin	0 gold coins	0 gold coins	100 gold coins
Balin	15 gold coins	45 gold coins	20 gold coins
Fili	10 gold coins	15 gold coins	15 gold coins
Kili	30 gold coins	20 gold coins	30 gold coins

a. Determine each dwarf's fair share (in gold coins).

Thorin's fair share:

Balin's fair share:

Fili's fair share:

Kili's fair share:

b. Determine which dwarf gets each item.

Mithril Shirt:

Gold Cup:

Arkenstone:

c. How many gold coins does each dwarf pay or receive in the end? *Show your work!*

Thorin:

Balin:

Fili:

Kili: