## 202401 Math F113X - Numbers and Society/Midterm 3

James Gossell Semester: Spring 2024

Date Exam Taken: \_\_\_\_\_

Print Your Name Clearly

Proctor's Name

Start Time (to be filled out by Proctor)

End Time (to be filled out by Proctor)

## Student Responsibilities:

- It is the student's responsibility to keep track of their time. Students are to complete the exam in one testing session.
- It is the student's responsibility to ensure all pages are included with the exam. The exam is **9** pages including this cover sheet.

Exam Specific Instructions:

- TIME LIMIT: 60 minutes
- Outside materials that are allowed: Calculators, writing utensils, and scratch paper are permitted.

Math F113X, Spring 2024

Name \_\_\_\_\_

Score \_\_\_\_\_

**1. (8 points)** Draw graphs that meet the following descriptions. (There are many possible answers.)

**a.** Draw a graph that is <u>not</u> connected.



**b.** Draw a graph that has a degree 6 vertex.



**c.** Draw a graph that has an Euler circuit.



**d.** Draw a graph that is a tree.



**2. (8 points)** Use Dijkstra's Algorithm to find the **shortest path from Vertex A to Vertex I**. *Please show your work, label the vertices, or explain how your know your answer is correct.* 



**3. (7 points)** The following diagram represents a subway system with 11 stations and 19 connections. Stacy is determined to start at station A and take all 19 connections exactly once with no repeats. *(Stacy is looking for an Euler path.)* 



**a.** Trace a route on the diagram which starts at **A** and crosses every edge exactly once.

b. No matter which route Stacy chooses, which station is she guaranteed to finish at?

**4. (12 points)** Auburn lives on the island village of Aysgarth, a lovely settlement consisting of 8 islands connected by 16 bridges. Auburn is determined to start and end at his house and cross every bridge exactly once. However, try as he may, Auburn cannot do it.

**a.** On his best attempt, Auburn crosses 15 of the 16 bridges. **Circle the bridge** on the diagram that Auburn was not able to cross.

**b. Draw a 17<sup>th</sup> bridge** so that Auburn will be able to cross all the bridges exactly once.

c. Trace a route for Auburn to cross all 17 bridges (the original 16 plus the one you drew).



**5. (15 points)** Pamela lives in Craig, Colorado and wants to travel to see family members in 3 different locations: Colorado Springs, Laramie, and Salt Lake City. Afterwards, she will return to Craig. A table of driving times between these cities is given below:

	Craig	Colorado Springs	Laramie	Salt Lake City
Craig	-	7 hours	6 hours	7 hours
<b>Colorado Springs</b>	7 hours	-	5 hours	11 hours
Laramie	6 hours	5 hours	-	8 hours
Salt Lake City	7 hours	11 hours	8 hours	-

Use the **Repeated Nearest Neighbor Algorithm** to help Pamela optimize her driving time. *For full credit, you must show all your work and give your final answer below.* 

Nearest Neighbor route starting/finishing at Craig:						
Total Time:	hours					
Nearest Neighbor route <b>sta</b>	rting/finishing at Colorado Springs:					
Total Time:	hours					
Nearest Neighbor route starting/finishing at Laramie:						
Total Time:	hours					
Nearest Neighbor route starting/finishing at Salt Lake City:						
Total Time:	hours					

**Final Answer:** Based on your work above, recommend the order of stops for Pamela as she visits her family members:

Start: <u>Craig</u> Stop 1: \_\_\_\_\_ Stop 2: \_\_\_\_\_ Stop 3: \_\_\_\_\_ End: <u>Craig</u>

6. (8 points) Peter the Postman has to deliver packages to 6 different locations. Being in a hurry, Peter decides to use the Sorted Edges Algorithm to reduce his travel distance as much as possible. The table below represents the distances (in miles) between all the locations. Use the Sorted Edges Algorithm to find a Hamiltonian circuit that minimizes Peter's travel distance. Draw lines to complete the Hamiltonian circuit on the diagram below. You do <u>not</u> need to find the total distance. You do <u>not</u> need to show your work.

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	Peter	Α	В	С	D	E
Peter	-	9	11	12	16	15
Α	9	-	10	8	14	15
В	11	10	-	14	13	15
С	12	8	14	-	7	6
D	16	14	13	7	-	4
E	15	15	15	6	4	-

**7. (7 points)** King Arthur wants to go on 10 different quests in various locations throughout Wales before returning his castle in Camelot. In order to minimize traveling time, his loyal knight Sir Percy suggests that he uses the Brute Force Algorithm.

**a.** How many possible Hamiltonian circuits are there is this application? (*Note: There would be 11 total vertices – 10 for the quests and 1 for the starting/ending location.*)

**b.** Arthur lives in the sixth century without modern computers or calculators. Would the Brute Force Algorithm be practical for Arthur to use? If not, which algorithm would you suggest for King Arthur? *Explain why.* 

**8. (10 points)** Rosie, Stacy, Tony, Wally, and Xavier are tree house neighbors. They all decide that they want to build bridges to connect their tree houses. They only have enough material to build bridges totaling 200 feet in combined distance.

**a.** What is the **minimum number of bridges** that the 5 friends will need to connect their tree houses (so they can all visit each other without stepping foot on the ground)?

**b.** Using the following diagram with distances listed below, find a **minimal cost spanning tree** of bridges that minimizes the total distance of the bridges. (Yes, I know... a spanning tree that spans the trees.) For full credit, **draw bridges on the diagram** which connect the tree houses.



**c.** What is the **total distance** between the bridges? Will the 5 friends be able to connect all their tree houses with 200 feet of materials?

**9. (12 points)** The table below gives a list of tasks with their corresponding times and prerequisites which are needed to complete a project.

Task	Time	Prerequisites
Α	2 hours	
В	4 hours	
С	6 hours	
D	6 hours	А
E	5 hours	А, В
F	3 hours	В, С
G	1 hour	С
Н	1 hour	D, E
I	2 hours	E <i>,</i> F
J	3 hours	F, G
К	1 hour	H, I, J

**a.** Draw a diagraph, clearly labeling each vertex with its corresponding time.

**b.** Create a priority list using the **critical path algorithm**. You do <u>not</u> need to create a schedule.

c. How long would it take to complete all these tasks with unlimited processors? Explain.

**10. (15 points)** Bruce and Tony are working on a series of tasks to build a special gauntlet. They want to finish it in 8 hours. They've labeled the tasks A, B, C, D, E, F, and G, and created a diagraph with the corresponding completion times in hours. The diagraph is given below:



- a. Create a priority list using the decreasing time algorithm.
- **b.** Using your priority list in part (a), **create a schedule for Bruce and Tony**.



c. With this schedule, will they be able to finish in 8 hours?

**d.** Was there any idle time in the schedule? Will it be possible for Bruce and Tony to finish faster with a different priority list?

**e.** If Bruce and Tony recruit their friends Steve and Natasha to help them with their project, will they be able to finish faster? *Write a sentence to explain your answer. Please mention "critical time" in your explanation.*