Spring 2025 Math F113X

Exam 1

Name: _____ **Section:** ☐ **10:30 am (Leah Berman)**
☐ **2:15pm (Jill Faudree)**

Rules:

- Partial credit will be awarded, but you must show your work.
- You may have 1/2 of a standard page of paper ($8.5'' \times 5.5''$ or $11'' \times 4.25''$) of notes, both sides.
- Calculators are allowed.
- Place a box around your $\boxed{\text{FINAL ANSWER}}$ to each question where appropriate.
- Turn off anything that might go beep during the exam.

Good luck!

Problem	Possible	Score
1	20	
2	12	
3	20	
4	12	
5	18	
6	18	
Extra Credit	(6)	
Total	100	

1. (20 points)

A certain borough in Alaska has switched to using **Instant Runoff Voting (Ranked Choice Voting)** to determine the winner of its mayoral races.

In a recent municipal election, the preference schedule for the race was as follows:

	47	24	15	10	14	10
1st choice	Sampson	Hopkins	Kassel	Sampson	Kassel	Ward
2nd choice	Hopkins	Kassel	Ward	Hopkins	Sampson	Kassel
3rd choice	Ward	Sampson	Sampson	Kassel	Ward	Sampson
4th choice	Kassel	Ward	Hopkins	Ward	Hopkins	Hopkins

a. How many voters voted in the election? _____

b. How many voters are needed to have a majority of the votes? _____

c. Was there a winner after round 1 (that is, before anyone was eliminated)? Why or why not? Explain your answer.

d. Was anyone eliminated in round 1? Explain your answer.

e. Determine the winner of the election. Show your work clearly, in a way that someone else can follow. If you require multiple rounds, show the computations clearly, and clearly state which candidate is eliminated.

The winner of the election was _____ after _____ rounds.

2. (12 points)

ASUAF is holding elections to decide who will represent UAF on the Board of Regents. There are four candidates (labeled A, B, C, and D for convenience). The preference schedule is in the table below.

number of voters	8	15	20	10	9	16
1st choice	A	B	C	D	A	B
2nd choice	C	D	A	C	C	D
3rd choice	D	C	D	A	B	A
4th choice	B	A	B	B	D	C

a. Find the winner under the plurality method. Show the calculations that give your answer.

b. Determine who would win if the only candidates were A and B. (That is, determine the winner in a head-to-head comparison of A and B.) Show the calculations that give your answer.

c. Based only on your calculation in part (b), **is it possible** for A to be the Condorcet Winner? Justify your answer.

d. Determine the point value **candidate C** would receive if the election were held using the Borda Count Method.

3. (20 points)

Consider the weighted voting system $[q : 10, 10, 5, 5, 5, 5, 1]$

1. What is the smallest value q can take? Justify your answer with a calculation.

2. Explain why there is no choice of q for which this voting system can have a dictator.

3. Suppose q is 36. So, the weighted voting system is $[36 : 10, 10, 5, 5, 5, 5, 1]$.

   (a) Identify any players with veto power or state that none exist. Justify your answer.

   (b) Identify any dummies or state that none exist. Justify your answer.

4. (12 points)

Consider the weighted voting system $[17 : 13, 9, 5, 2]$

a. Determine all winning coalitions with one or two players. List of them in the space below (b).

b. All winning coalitions using 3 or 4 players are listed below. Underline the players that are critical in each coalition (both the 2 player and the 3 or more players coalitions). Then find the Banzhaf power distribution for this system.

winning coalitions with 1 or 2 players	winning coalitions with 3 or more players
	P_1, P_2, P_3
	P_1, P_2, P_4
	P_1, P_3, P_4
	P_1, P_2, P_3, P_4

c. **Based on your calculations in part (b)**, does this system contain any dummy players? Justify your conclusion.

5. (18 points)

Amanda and Bernadette pooled their money to buy a fancy box of handmade valentine's day heart-shaped truffles. The candy company makes boxes of truffles that contain three flavors of filling: caramel, raspberry, and hazelnut. Each box of candy contains 12 truffles, 3 each of caramel and raspberry and 6 of hazelnut. (See box right.) The box of truffles costs $24.

c	c	c	h	h	h
r	r	r	h	h	h

a. What is the dollar value of a fair share? _____

b. Amanda will not eat caramel, and she likes raspberry twice as much as hazelnut.

(i) What is the value of **each** truffle to her?

single caramel: _____ single raspberry: _____ single hazelnut: _____

(ii) Which of the following collections of truffles are a fair share for Amanda (if any)? Circle the answer, and write in the total value for Amanda.

	c c c h	c c c h r r h h	r r r
Value			
Fair?	yes no	yes no	yes no

c. Bernadette values a single caramel truffle at $1, a single raspberry truffle at $5, and a single hazelnut truffle at $1.

(i) If Bernadette is the divider, show a division of the box of candy that Bernadette might make, and explain why.

c	c	c	h	h	h
r	r	r	h	h	h

(ii) Which portion of chocolates would Amanda choose? Why? What is the total value to her of that portion?

6. (18 points)

Alexis, Jamal, and Kasey buy a small cabin. They decide to divide the time each one occupies the cabin using the lone divider method. One person is chosen to be the divider and divides the year into 3 parts that have equal value to the divider. The table below represents the value of each section in each person's eyes.

	Jan-May	**Jun-Sept**	**Oct-Dec**
Alexis	\$25,000	\$25,000	\$25,000
Jamal	\$30,000	\$27,000	\$18,000
Kasey	\$33,000	\$30,000	\$12,000

a. Identify who the divider was. _____

b. Circle the values in the table that represent a fair share to each person.

c. What happens in the next step of the lone divider process? Explain.

d. Suppose they instead had submitted the following table of values.

	Jan-May	**Jun-Sept**	**Oct-Dec**
Alexis	\$25,000	\$25,000	\$25,000
Jamal	\$5,000	\$70,000	\$5,000
Kasey	\$1,000	\$73,000	\$1,000

What happens in the next step of the lone divider process? Explain what the next step in the process should be and why (but you don't have to actually implement the step).

7. (Extra Credit: 6 points)

Andrea and Zeke are dividing up three items: an espresso maker, a cleaning robot, and a microwave. They submit the following sealed bids for the three items.

	Espresso Maker	Cleaning robot	Microwave
Andrea	\$60	\$120	\$30
Zeke	\$100	\$50	\$75

a. Determine each person's fair share (in dollars).

Andrea's fair share	Zeke's fair share

b. Determine which person gets each item.

Espresso Maker	Cleaning Robot	Microwave

c. Determine the **surplus**. Show your work!

d. How many dollars does each person pay or receive in the end? Show your work!

Andrea:

Zeke: