

Spring 2025

Math F113X

Exam 3

Name: _____

Section: ☐ 10:30 am (Leah Berman)
☐ 2:15pm (Jill Faudree)

Rules:

- Partial credit will be awarded, but you must show your work.
- You may have 1/2 of a standard page of paper (8.5" × 5.5") of notes, both sides.
- Calculators are allowed.
- Place a box around your **FINAL ANSWER** to each question where appropriate.
- Turn off anything that might go beep during the exam.
- A formula sheet (tabula recta, finance formulas) is included on the last page of the exam. You may tear it off if you need to.

| Problem | Possible | Score |
|--------------|----------|-------|
| 1 | 12 | |
| 2 | 16 | |
| 3 | 10 | |
| 4 | 16 | |
| 5 | 5 | |
| 6 | 5 | |
| 7 | 20 | |
| 8 | 6 | |
| 9 | 10 | |
| Extra Credit | (5) | |
| Total | 100 | |

1. (12 points)

- a. Encrypt the message EAGLE using an alphabetic Caesar cipher with shift 8 (mapping A to I).

- b. Decrypt the message AYCQL if it was encrypted using an alphabetic Caesar cipher with shift 8 (mapping A to I).

2. (16 points)

- a. Encrypt the message SPRING IS HERE using a tabular transposition cipher with the encryption keyword JUMBO.

- b. Decrypt the message NSEZM ENROM ZSBAB BOADA if it was encrypted using a tabular transposition cipher with the encryption keyword JUMBO.

3. (10 points)

Pick **ONE** of the following. Make sure to clearly identify which one you want graded by checking the appropriate box.

Briefly explain the technique you used to decrypt the message.

- a. (Grade this one: ☐) Decrypt the message VFBLV if it was encrypted using a **shifting Caesar cipher** that started with a shift of 4 (mapping A to E).
- b. (Grade this one: ☐) Decrypt the message MECZD if it was encrypted using a **Vigenère cipher** with keyword FACE.

4. (16 points)

- a. Which of the encryption methods below are vulnerable to being decrypted using frequency analysis? Check the box to indicate it is vulnerable.

☐ Caesar cipher ☐ tabular transposition ☐ Vigenère

☐ shifting Caesar cipher ☐ double transposition

- b. Below is a table defining a **randomly assigned** substitution mapping. Describe one advantage of this encryption mechanism and one disadvantage.

| | | | | | | | | | | | | | | | | | | | | | | | | | | |
|----------|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| original | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
| maps to | E | M | S | U | N | H | T | I | P | J | O | C | D | R | A | Y | V | B | Z | W | F | X | Q | G | K | L |

5. (5 points)

You want to encrypt the message YOUR COVER IS BLOWN. LEAVE SAFE HOUSE NOW. using a Vigenère cipher. Pick a code word and explain why it is a good choice. Note: You are **not** asked to encrypt the message!

6. (5 points)

Your bill at a restaurant is \$38.67 and you want to leave an 18% tip. How much would you add to your bill? Show your calculation.

7. (20 points)

For each scenario below, **identify** the formula you should use and then **plug appropriate numbers into** that formula. You do **not** need to calculate the the value.

- a. You loan your friend \$400. They agree to pay an annual interest rate of 5% simple interest. Eighteen months later, they repay the loan. How much did they pay you?

- b. You deposit \$1000 in an account that earns an annual interest rate of 4.8% APR. The interest is compounding weekly. How much will the account be worth in 10 years?

- c. You want to take out a loan to buy a \$150,000 home. The bank offers a 30-year mortgage with an interest rate of 6.2%. What will the monthly payments be? (Suppose the interest is compounded monthly.)

- d. You deposit \$4000 in an account the earns 2.75% APR compounded daily for 7 years. How much interest did you earn?

8. (6 points)

It is a fact that

$$441,488 = 10,000 \left(1 + \frac{0.05}{12} \right)^{(12) \cdot (42)}.$$

Suppose this calculation is used to model a savings account. Explain what this calculation indicates about the account parameters.

- a. How much was invested at the start? _____
- b. What was the annual interest rate? _____
- c. How frequently is the interest being compounded? _____
- d. How long was the money invested? _____
- e. Explain, in your own words, what the number 441,488 represents.

9. (10 points)

Suppose you charge \$2000 on a credit card with an annual percentage rate of 29% compounded monthly and you plan to make monthly payments of \$40.

- a. How much do you owe at the end of the first month? (Show your computation, and actually compute it.)
- b. What does the calculation in part (a) indicate about when you will pay off the loan?

Extra Credit (5 points)

Pick **one** of the two options. Indicate which one you want graded.

(a) (Grade this one: \square)

Suppose you have the following part of a spreadsheet, where ### represents numbers that have been entered in (but that could be changed).

| | A | B | C | D | E |
|---|-----|-----|---|---|---|
| 1 | P | r | n | t | A |
| 2 | ### | ### | 1 | 1 | |
| 3 | | | | 2 | |
| 4 | | | | 3 | |
| ⋮ | | | | ⋮ | |

- i. Write a formula, using cell references (A1, B2, C3, etc.), to put into cell E2 to compute the total amount of money at the end of the first year.

- ii. You want to drag cell E3 down to automatically compute compound interest (annually compounded) for subsequent years. How would you change your previous formula to be able to do that?

(b) (Grade this one: \square)

Decrypt the text

S2L53 CDIR8 TEOE4 R3H

if it was encrypted using a double transposition cipher with first keyword JUMBO and second keyword KEYS.

Formulas

$$A = P + I$$

$$A = P(1 + rt)$$

$$A = P\left(1 + \frac{r}{n}\right)^{(nt)}$$

$$P = \frac{A}{\left(1 + \frac{r}{n}\right)^{(nt)}}$$

$$P = \frac{d(1 - \left(1 + \frac{r}{n}\right)^{(-nt)})}{\left(\frac{r}{n}\right)}$$

$$d = \frac{P\left(\frac{r}{n}\right)}{\left(1 - \left(1 + \frac{r}{n}\right)^{(-nt)}\right)}$$

| | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
| B | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A |
| C | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B |
| D | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C |
| E | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D |
| F | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E |
| G | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F |
| H | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G |
| I | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H |
| J | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I |
| K | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J |
| L | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K |
| M | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L |
| N | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M |
| O | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N |
| P | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O |
| Q | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P |
| R | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q |
| S | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R |
| T | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S |
| U | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T |
| V | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U |
| W | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V |
| X | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W |
| Y | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X |
| Z | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y |