Spring 2026 **Math F113X**

Exam 1

Name: Solutions **Instructor:** _____

Rules:

- Partial credit will be awarded, but you must show your work.
- You may have a 3in $\times$ 5in notecard with writing on both sides.
- Calculators are allowed.
- Turn off anything that might go beep during the exam.

Good luck!

Problem	Possible	Score
1	16	
2	8	
3	10	
4	10	
5	12	
6	12	
7	10	
8	12	
9	10	
Extra Credit	(4)	
Total	100	

1. (16 pts. total) Consider the following preference schedule for an election with 4 candidates and 100 voters.

	24	10	21	27	18
1st	A	B	~~D~~	C	B
2nd	C	A	A	~~D~~	A
3rd	~~D~~	C	C	A	C
4th	B	~~D~~	B	B	~~D~~

(a) (3 pts.) Who wins a plurality vote in this election? Show your work.

Vote Tally:

A	B	C	D
24	28	27	21

B wins

(b) (8 pts.) If instead an IRV (ranked choice) election was held, the first round would not produce a winner, but D would be eliminated. Knowing this, determine the outcome of the **second round**, showing your work. Give the winner (if there is one), or say who is eliminated next.

New Tally without D

A: 24 + 21 = 45

B: 10 + 18 = 28

C: 27 = 27

Still no majority.

No winner.

C is out

Candidate who wins in second round: X OR Candidate eliminated after second round: C

(c) (5 pts.) If opinion polls before the election showed A was running behind several other candidates, the voters ranking A first might have decided to instead vote for their second choice. Would this action have changed the outcome? Who would have won?

The 24 votes for A would go to C.

New Vote Tally

B: 10 + 18 = 28

C: 24 + 27 = 51

D: 21 = 21

C would win with a majority.

2. (8 pts.)

A Borda Count will be used to determine the winner in the election with voting schedule on the right.

	2	5	7	points
1st	A	B	C	3
2nd	C	C	B	2
3rd	B	A	A	1

(a) (4 pts) Showing your work, calculate the weighted vote for candidate B.

B: $2(1) + 5(3) + 7(2)$

$2 + 15 + 14 = 31$

(b) (2 pts) If the weighted vote for candidate A is 18 and for candidate C is 35, who is the Borda winner?

C, it has the largest sum

(c) (2 pts) Is the Borda winner also a majority winner? Explain you reasoning.

total number of votes: $2+5+7=14$; votes for C: 7

No. To be a majority, C needed at least 8 votes.

3. (10 pts. total)

In deciding among four candidates for promotion, Abby (A), Burt (B), Camile (C), and David (D), six members of the senior management rank their preferences as shown, and then compile the following table of outcomes from most of the head-to-head match-ups.

	2	1	3
1st	A	B	C
2nd	B	C	A
3rd	C	D	B
4th	D	A	D

Match up	A vs. B	A vs. C	A vs. D	B vs. C	B vs. D	C vs. D
Winner	A	C	A	tie	B	C

(a) (3 pts.) Who would win the A vs. C head-to-head vote? Show your work.

Vote tally A vs. C: A: 2

C: 1+3=4

C wins

(b) (3 pts.) Is there a Condorcet winner? Explain why or why not.

No. Each candidate loses to or ties at least one other candidate.

A loses to C, B loses to A, C ties with B, D loses to C.

(c) (4 pts.) Who would win by Copeland's method? Show work to support your answer.

Copeland's Vote tally: A: $1+1 = 2$

B: $\frac{1}{2}+1 = 1.5$

C: $1+\frac{1}{2}+1 = 2.5$

D: $= 0$

C wins

4. (10 pts) Consider the weighted voting system $[18 : 8, 4, 4, 3, 3, 2]$. Find:

   (a) (2 pts) The total number of players: 6

   (b) (2 pts) The total number of votes: $8+4+4+3+3+2=24$

   (c) (2 pts) The quota: 18

   (d) (4 pts) Identify which players have veto power. Show a calculation that justifies your conclusion.

   P_1 has veto power.

   Weight of remaining players: $4+4+3+3+2=16<18$

5. (12 pts) Consider the weighted voting system $[q : 9, 4, 2]$.

   (a) (3 pts) What is the smallest possible quota value?

   $9+4+2=15$; $\frac{15}{2}=7.5$. So $q=8$ is the smallest.

   (b) (4 pts) Find **all** possible values of q that result in a dictator. **State which player** is the dictator and **explain** why this is the case.

   $q=8$, $q=9$. In both cases, P_1 is a dictator because their weight is at least the quota.

   (c) (5 pts) Find **all** possible values of q that result in one or more dummies? **State which player(s)** are dummies and **explain** why this is the case.

   $q=8, q=9$: dummies are P_2 and P_3 because P_1 is a dictator

   $q=12, 13$: P_3 is a dummy since neither P_1P_3 or P_2P_3 are winning coalitions.

6. (12 pts) Consider the weighted voting system $[29 : 17, 13, 12, 4]$. A list of all seven **winning** coalitions are listed below. The critical players have been underlined in each winning coalition **except for the two in boxes.**

All Winning Coalitions

underlines: 12

$\underline{P_1}, \underline{P_2}$	$\underline{P_1}, P_2, P_3$	$\underline{P_2}, \underline{P_3}, \underline{P_4}$	P_1, P_2, P_3, P_4
$\underline{P_1}, \underline{P_3}$	$\underline{P_1}, \underline{P_2}, P_4$	$\underline{P_1}, \underline{P_3}, P_4$	

(a) (3 pts) Provide a calculation with explanation demonstrating that the coalition P_1, P_3, P_4 is a winning coalition.

weight of P_1, P_3, P_4 is $17+12+4 = 33 > 29 = q$

(b) (4 pts) **For the two winning coalitions in boxes,** underline any players that are critical. Explain your reasoning below.

For P_1, P_3, P_4, we know $P_1 P_3$ is winning so P_4 is not critical.

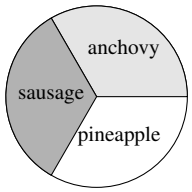
For $P_1 P_2 P_3 P_4$, no player is critical because any one can leave and the coalition will still win.

(c) (5 pts) Find the Banzhaf power distribution for this system. Give each player's power as a fraction.

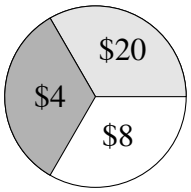
player	P_1	P_2	P_3	P_4
Banzhaf Power Index	$\frac{5}{12}$	$\frac{3}{12}$	$\frac{3}{12}$	$\frac{1}{12}$

quick check: $5+3+3+1 = 12$ ✓

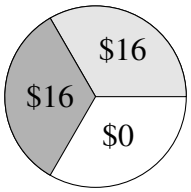
7. (10 pts) Tim and Harold decide to split a pizza with three separate toppings using the **Divider-Chooser** method. The total cost of the pizza is \$32. Below is a picture of the pizza and pictures indicating how Tim and Harold value the parts of the pizza.



Tim



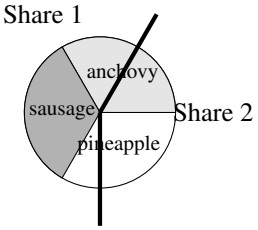
Harold



(a) (2 pts) How much value is a fair share of the pizza? \$ $\frac{32}{2} = \$16$

(b) Tim is selected to be the divider and splits the pizza into two shares indicated below in a picture and a table.

Share 1:	all of the sausage, 1/2 of the anchovy, 1/4 of the pineapple
Share 2 :	1/2 of the anchovy, 3/4 of the pineapple

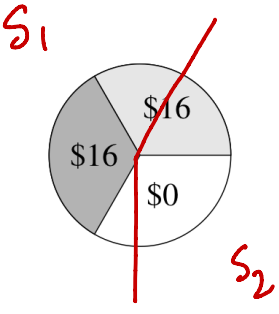


i. (4 pts) Provide a calculation that demonstrates Share 2 is a fair share to Tim.

Value of Share 2:

$\frac{1}{2}(\$20) + \frac{3}{4}(\$8) = \$10 + \$6 = \$16$ ✓

ii. (4 pts) Which share would Harold choose? Circle the correct answer and provide a calculation to support your answer.



Share 1

\$16 + \$8 + 0 = \$24

Share 2

\$8

← Harold's Value

8. (12 pts) Tim, Harold, and Janet will split several bags of coffee worth \$90 using the **Lone Divider** method. One of the three friends is selected to divide the coffee into three shares. The values are shown below.

Person	Share 1	Share 2	Share 3
Tim	\$40	\$20	\$30
Harold	\$30	\$30	\$30
Janet	\$60	\$15	\$15

(a) (2 pts) What is the value of a fair share to each person?

$\frac{\$90}{3 \text{ people}} = \$30/\text{person}$

(b) (3 pts) Who was the divider? How can you tell?

Harold. All shares are equal.

(c) (2 pts) **Circle the bids (dollar values) in the table** that are fair shares to each person.

(d) (5 pts) Is it possible to allocate the shares so that everyone gets a fair share? If so, who gets which share? If not, what happens next in the lone divider process?

Tim : Share 3
Harold : Share 2
Janet : Share 1

It is possible to allocate all shares.

9. (10 pts) Yelena and Zach are going to divide their shared kitchen gadgets, a coffee maker and a microwave, using the method of sealed bids. Fill in the blanks below.

	Yelena	Zach
coffee maker	\$100	\$90
microwave	\$50	\$70
total bid	\$150	\$160
(a) (2 pts) What is a fair share for each person?	$\frac{150}{2} = \$75$	$\frac{160}{2} = \$80$
award (who gets what item)	coffee maker	microwave
award value	\$ 100	\$ 70
(b) (4 pts) How much money does each person pay in or receive from the holding pile? Make sure to indicate whether the person pays in or receives.	$100 - 75 = 25$ pays in \$25	$70 - 80 = -10$ receives \$10
(c) (2 pts) Determine the surplus.	$25 - 10 = \$15$	
(d) (2 pts) Determine each person's fair share of the surplus.	$\frac{15}{2} = \$7.50$	\$7.50

Extra Credit: (4 pts) Determine the Final Allocation by filling out each blank below.

Summary of Final Allocation			
person	items	cash	paid or received?
Yelena	coffee maker	\$17.50	paid
Zach	microwave	\$17.50	received