Spring 2026 **Math F113X**

Exam 2

Name: Solutions **Instructor:** _____

Rules:

- Partial credit will be awarded, but you must show your work.
- You may have a 3in $\times$ 5in notecard with writing on both sides.
- Calculators are allowed.
- Turn off anything that might go beep during the exam.

Good luck!

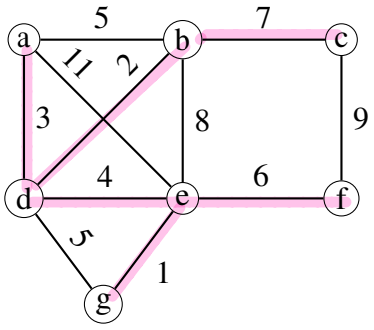
Problem	Possible	Score
1	12	
2	12	
3	12	
4	12	
5	12	
6	10	
7	15	
8	15	
Extra Credit	(4)	
Total	100	

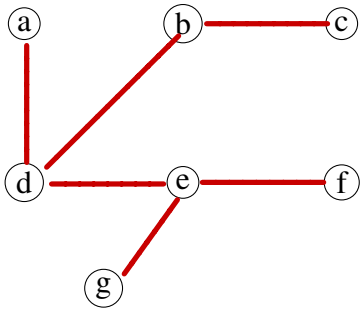
1. (12 pts. total)

   (a) (6 pts.) Perform Kruskal's algorithm to find a minimum weight spanning tree for the following graph. Draw your tree, connecting the vertices provided, and determine the total weight.

edge	weight	used?
eg	1	✓
bd	2	✓
ad	3	✓
de	4	✓
ab	5	✗
dg	5	✗
ef	6	✓
bc	7	✓
be	8	
cf	9	
ae	11	

done





Total weight of spanning tree: $1+2+3+4+6+7=23$

   (b) (6 pts.) Describe one real-world situation in which it would be useful to find a minimal spanning tree for a graph, including what the vertices, nodes, and edge weights in the graph represent.

Vertices represent cities

Edge represent electricity lines between cities

Edge weights represent the cost of fixing them

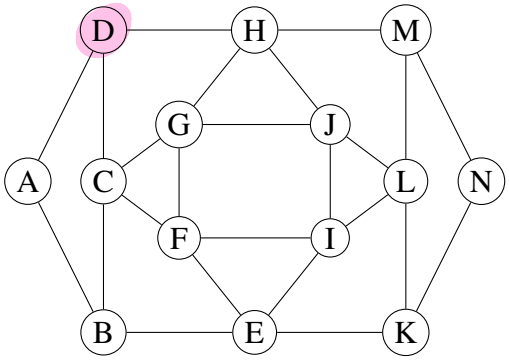
Reason to find a minimal spanning tree:

It's the cheapest way to make sure all cities are on the electrical grid.

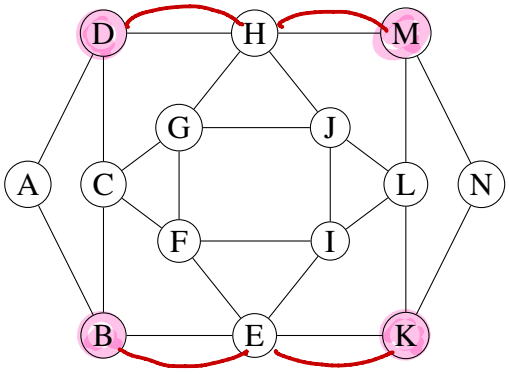
2. (12 pts total)

(a) (3 pts.) Explain why the graph on the right does **not** have an Euler circuit.

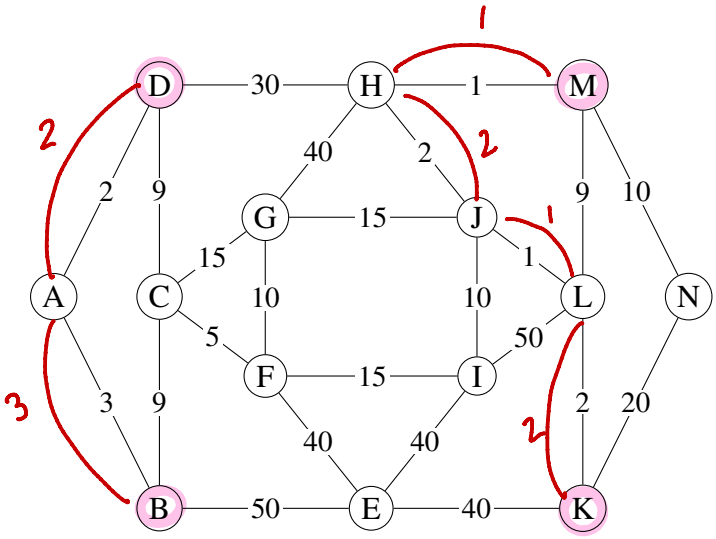
The vertex D has odd degree



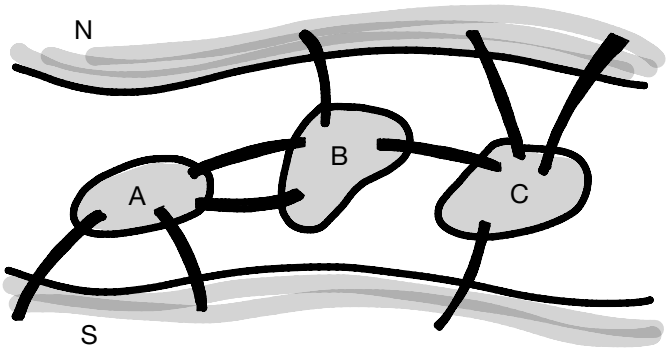
(b) (5 pts.) **Duplicate** the **fewest** number of edges such that the resulting graph will have an Euler circuit. Note: You do not have to find the circuit.



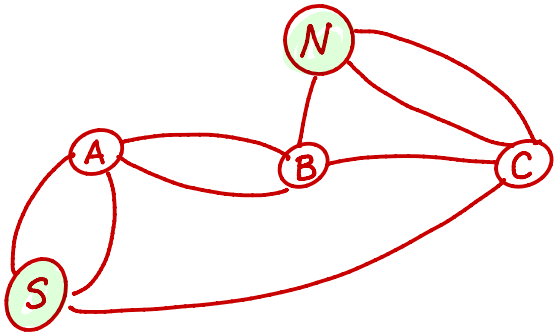
(c) (4 pts.) Suppose the edges are now weighted (see below). Duplicate edges such that the resulting graph will have an Euler circuit of **smallest possible total weight.**



3. (12 pts. total) There is a river running through the middle of a city with three islands and nine bridges, shown in the figure below.



(a) (6 pts.) Draw a graph below that models this scenario.



(b) (6 pts.) A tourist wants to travel from the South Bank (S) to the North Bank (N) going across every bridge **at least** one time. Suppose there is a cost of $2.00 every time one crosses a bridge. What is the cheapest **achievable** cost of such a trip? Justify your answer.

cost: $\$2 \cdot 9 = \$18$

justification: The graph has two vertices of odd degree, S and N. So it has an Eulerian circuit. So it is possible to start in S, go across every edge exactly one time, and finish at N.

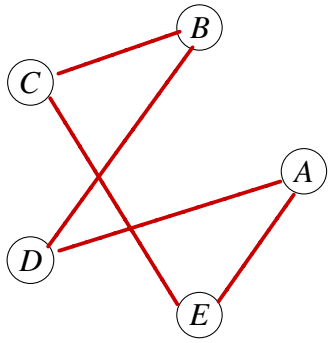
4. (12 pts. total) A company plans to run a bus continually throughout the day in a loop around 5 tourist stops in a city, denoted A, B, C, D, and E. The company would like to find a route that minimizes the time to complete a loop. Driving times are shown in the table.

	A	B	C	D	E
A	-	4	7	3	2
B	4	-	5	6	9
C	7	5	-	8	6
D	3	6	8	-	5
E	2	9	6	5	-

(a) (3 pts.) **Using terminology from Graph Theory**, what is the company looking for?

A minimum weight hamiltonian circuit.

(b) (6 pts.) Use the Sorted Edges / Cheapest Link algorithm to find a good route for the bus. A table of driving times in increasing order is given. You may draw your route connecting the vertices provided if you wish, but be sure to also give your answer as a list of vertices, along with the total time on the bus.



edge	time	used?
AE	2	✓
AD	3	✓
AB	4	✗
BC	5	✓
DE	5	✗
CE	6	✓
BD	6	✓
AC	7	
CD	8	
BE	9	

done

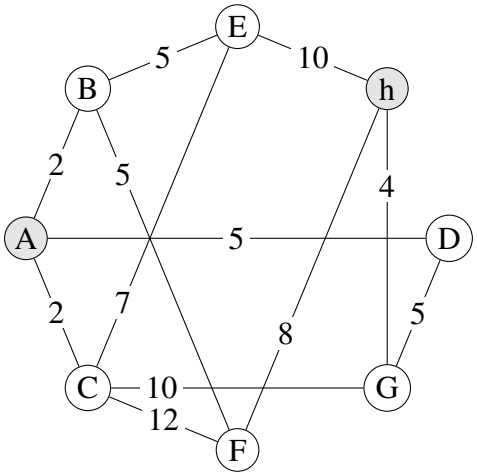
Route (list vertices) A E C B D A

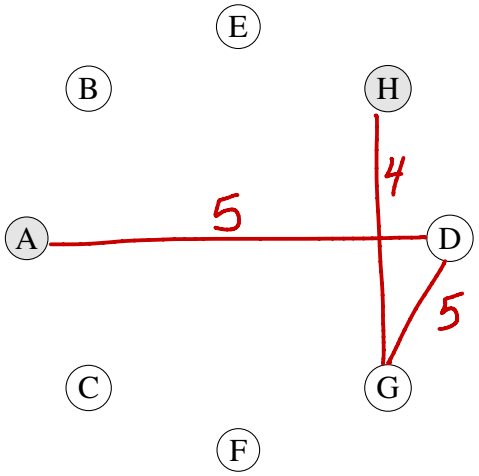
Total time on bus 2+3+5+6+6 = 22

(c) (3 pts.) What other algorithm might you use for this problem if you were not confident that you had found the best route?

Nearest neighbor.

5. (12 pts. total) Answer the following questions about the graph drawn below. A copy of the vertices are provided to use as scratch work if desired.





(a) (4 pts.) Find a shortest path from vertex A to vertex H. State the vertices on the path and its total weight. You are not required to use any particular method.

shortest path: A D G H total weight: 14

(b) (8 pts.) Describe one real-world situation for which finding a shortest path would be useful.

i. The vertices represent cities

ii. There is an edge between two vertices if there is a road between them

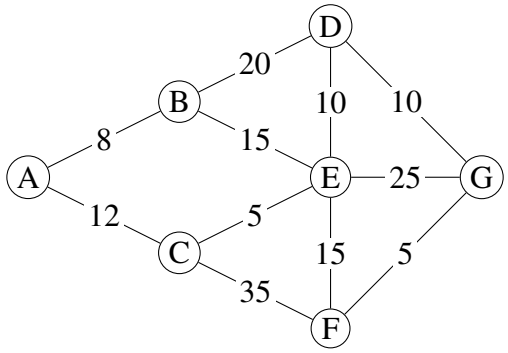
iii. The weight of edge represents the time to drive that section of road

iv. Reason to find a shortest path:

It's the fastest route to take to get from city A to city H.

6. (10 pts. total) In this question you will be asked to do **only a few steps** Dijkstra's Algorithm.

At no point are you asked to complete Dijkstra's Algorithm from beginning to end.

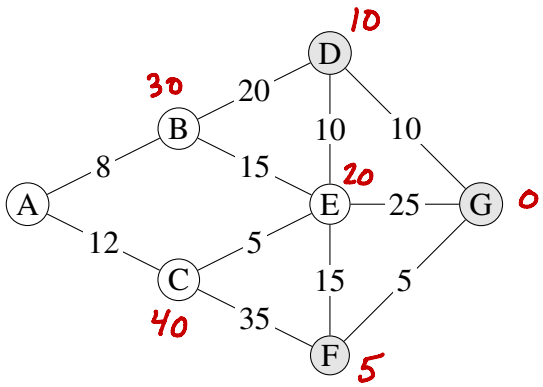


(a) (5 pts.) Begin Dijkstra's Algorithm, showing your work in the table below. **STOP** when you mark the **second** vertex as current.
DO NOT GO PAST THIS POINT. DO NOT DO THE WHOLE ALGORITHM.

vertex	current/ visited	**tentative** minimum distance to G	preceding vertex
A			
B			
C			
D		10	G
E		25	G
F		5	G
G	~~C~~ V	0	—

(b) (5 pts.) A few more steps of Dijkstra's Algorithm have been completed for you. Identify the **next current vertex** and update appropriate rows in the table below. **STOP** once your current vertex is marked as **visited**. ↑ the

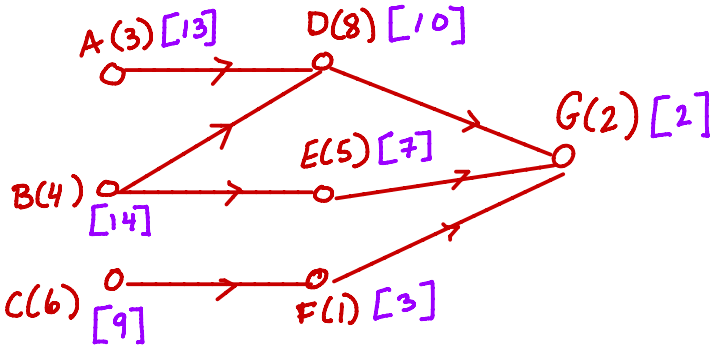
vertex	current/ visited	**tentative** minimum distance to G	preceding vertex
A			
B		30	D
C		~~40~~ 25	~~F~~ E
D	~~C~~, V	10	G
E	~~C~~, V	~~25~~ 20	~~G~~ F
F	~~C~~, V	5	G
G	~~C~~, V	0	–



7. (15 pts total) Consider the following table of tasks and dependencies.

Task	Time	dependency
A	3 hours	
B	4 hours	
C	6 hour	
D	8 hours	A,B
E	5 hour	B
F	1 hours	C
G	2 hour	D,E,F

(a) (5 pts.) Construct a **scheduling digraph** corresponding to the tasks and dependencies above.



(b) (2 pts.) What priority list do you get if you prioritize the tasks using the **Decreasing Time Algorithm**?

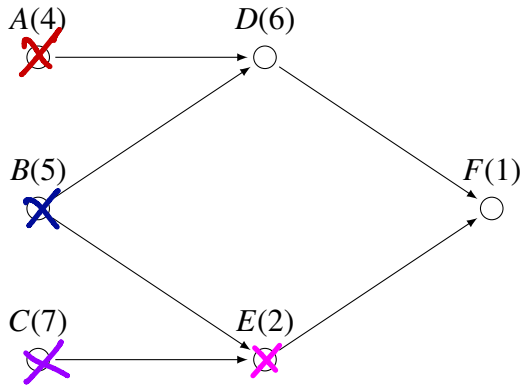
D	C	E	B	A	G	F
8	6	5	4	3	2	1

(c) (5 pts.) Label the vertices in your scheduling digraph using the **Backflow Algorithm**. **Use square brackets** – [ ] – for these labels.

(d) (3 pts.) What priority list do you get if you prioritize the tasks using the **Critical Path Algorithm**?

B	A	D	C	E	F	G
14	13	10	9	7	3	2

8. (15 pts) Consider the following digraph. The units for the tasks are **hours**.

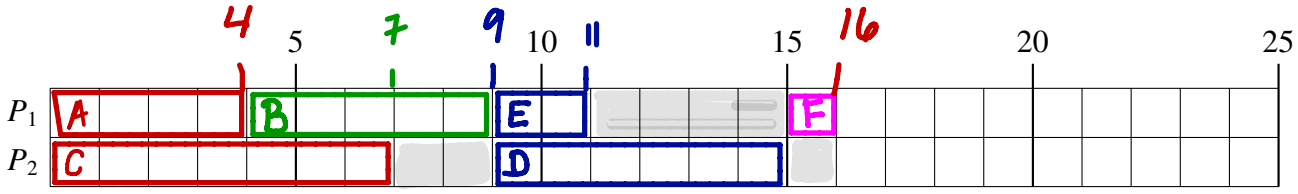


(a) (10 pts) Construct a schedule for two processors using the **priority list**

E, A, D, F, C, B

Make sure to label the tasks in the schedule. You may use the table below to track your work.

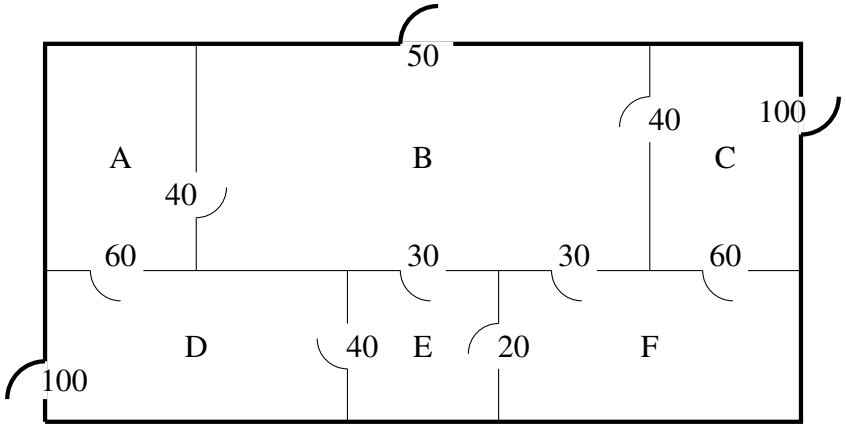
time	0	4	7	9	11	15
done		A	C	B	E	D
ready	A B C	B	—	D E	—	F



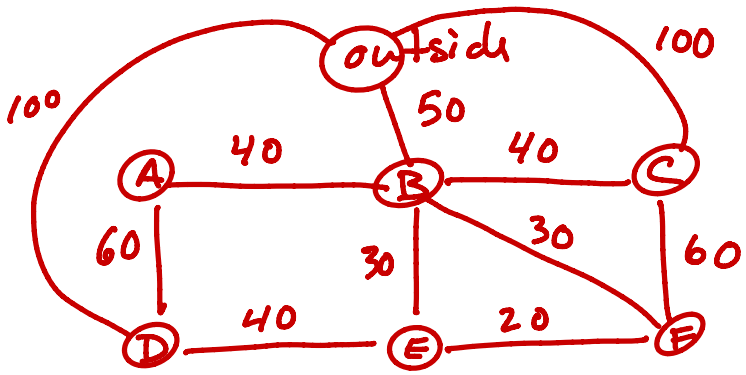
(b) (3 pts) How much **idle time** is in this schedule? 7 hours

(c) (2 pts.) How long did this schedule take to complete? 16 hours

Extra Credit (4 pts.) Below is the floor plan of an art gallery with 6 galleries and 11 doorways. A security company has assigned a theft risk assessment for each doorway. The higher the number the greater the risk.



(a) Draw a graph below that models this scenario.



(b) The gallery owners want all rooms to be available to visitors but they would like to shut and lock as many doors as possible to lower their risk. **Using terminology from Graph Theory**, what is the company looking for?

A minimum weight spanning tree