Math F113X: Homework Set 7

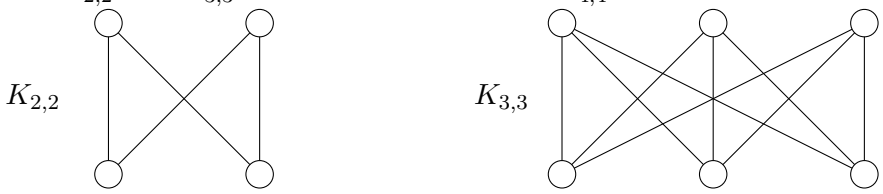
- Start with the introductory Problems A, B, and C.
- Then complete problems #19ac, 21, and 22ac from the Graph Theory section.
- Next, complete Problems D and E.
- Finally, answer the following **reflection question**: What did you learn from checking your homework answers against the provided solutions?

Problem A:

1. Write out what 5! means and find its value.
2. Expand 8.75×10^8
3. Write $35,200,000$ in scientific notation.
4. Go to WolframAlpha to compute 20! and write it in scientific notation, holding only the first three digits.

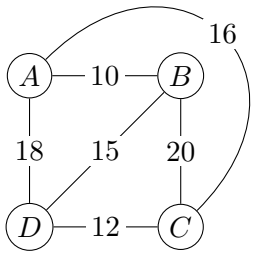
Problem B: This question is about notation of complete graphs and complete bipartite graphs.

1. We use K_n to denote the **complete graph** on n vertices. Draw K_3, K_4, K_5, and K_6.
2. We use $K_{n,n}$ to denote the **balanced complete bipartite graph** on $2n$ vertices. Observe that $K_{2,2}$ and $K_{3,3}$ are drawn below. Draw $K_{4,4}$.



3. How many edges would K_{10} have? How many edges would $K_{10,10}$ have?
4. Give an example of a practical situation that could be modeled by a weighted $K_{10,10}$? (State what the vertices and edge weights represent.)

Problem C: Answer the questions about the graph below.



1. How many different Hamiltonian circuits are possible? (Assume that it doesn't matter what the starting vertex is.)
2. List all of Hamiltonian circuits and calculate their weight.
3. Using the list above, identify a circuit with the lowest total weight.
4. The steps above are those of what algorithm?
5. Using the work above, find a Hamiltonian *path* of lowest total weight.

Problem D: Suppose someone wants to visit the capital city of every state in the contiguous 48 states and Washington DC. So, they will visit 49 cities in total. [NOTE: you will need to use a computational tool, like WolframAlpha, to complete this problem.]

1. Suppose they want to start and end the 49-city-tour at the same place. How many different tours are possible? (Your answer should be in *both* factorial notation and in scientific notation.)
2. Suppose you have a computer that can calculate the length of 1000 49-city-tours in one second. How long would it take the computer to calculate the length of all possible 49-city-tours? Give your answer in *years*.
3. What does your answer in part 2 indicate about the Brute Force algorithm?

Problem E: Create weighted, 5-vertex graph with vertex set A, B, C, D, and E, such that the Nearest Neighbor algorithm starting at A will give an optimal solution but starting at B will give the worst possible solution. Show your answer is correct.

Remember to write up your homework solutions according to the homework writeup guidelines.

Homework is graded using the following rubric for each problem (or problem part):

2 points: You provided a complete answer, with supporting work, written up clearly

1 point: Some attempt at a solution, but incomplete writeup / unclear / illegible / no answer

1 point: Only an answer, with no supporting work

0 points: Missing.

After you do the homework, you need to check your answers against the solutions! Then figure out your errors (if any) and revise your homework before you submit it. Finally, answer the reflection

question.

Homework must be submitted on Gradescope.