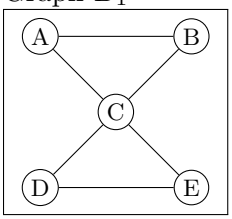
Math F113X: Homework Set 6

Answer the following problems from the Graph Theory section in the order assigned.

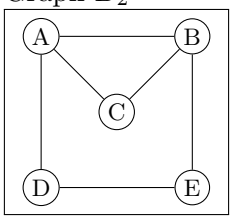
- problems on Euler circuits, paths, eulerization: A, 2, 14, 30, 32, 35a
- problems on Hamiltonian circuits and paths: B, C, D, 17, 18

Problem A: For each graph below, (i) determine if it has an Euler circuit and (ii) determine if it has an Euler path. Justify your answer.

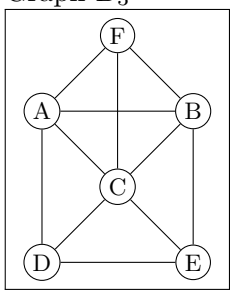
Graph B_1



Graph B_2



Graph B_3

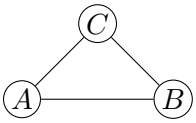


Problem B:

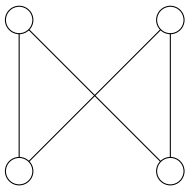
1. Write out what 5! means and find its value.
2. Expand 8.75×10^8
3. Write $35,200,000$ in scientific notation.
4. Use any computational device to compute 20! and write it in scientific notation, holding only the first three digits.

Problem C: This question is about **complete graphs** and **complete bipartite graphs**.
A complete graph on n vertices is the graph that includes an edge between every pair of distinct vertices, denoted K_n. A complete balanced bipartite graph on $2n$ vertices, denoted $K_{n,n}$, is the

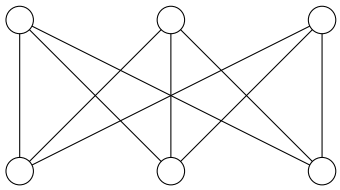
graph that splits the vertex set into two equal sets and includes every edge between the two sets. Examples are below.



K_3



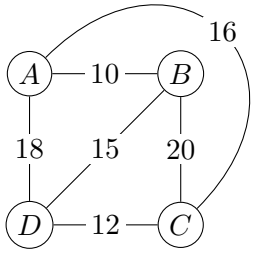
$K_{2,2}$



$K_{3,3}$

1. Draw K_4, K_5, and K_6.
2. Draw $K_{4,4}$.
3. How many edges would K_{10} have? How many edges would $K_{10,10}$ have?
4. Give an example of a practical situation that could be modeled by a weighted $K_{10,10}$? (State what the vertices and edge weights represent.)

Problem D: Answer the questions about the graph below.



1. List all of Hamiltonian circuits and calculate their weight.
2. Using the list above, identify a circuit with the lowest total weight.
3. The steps above are those of what algorithm?
4. Find a Hamiltonian path of lowest total weight, and justify your conclusion.

Remember to write up your homework solutions according to the homework writeup guidelines.

Homework is graded using the following rubric for each problem (or problem part):

2 points: You provided a complete answer, with supporting work, written up clearly

1 point: Some attempt at a solution, but incomplete writeup / unclear / illegible / no answer

1 point: Only an answer, with no supporting work

0 points: Missing.

After you do the homework, you need to check your answers against the solutions! Then figure out your errors (if any) and revise your homework before you submit it.

Homework must be submitted on Gradescope.