Goal: Understand what is meant by **a fair share** and how to calculate it.

1. The context of Fair Division

- We have some stuff to divide among parties
  - company dividing assets
  - head of family dividing an estate
  - roommates dividing chores/stuff
- It has a total value everyone can agree on
- Different people value stuff differently

Assumptions

1) There is no communication between parties (think: contention!)
2) No knowledge of each party's valuation
3) Everyone can be rational (no emotion)
4) No outside arbitration: parties have to work things out among themselves!

2. The definition of **a fair share**:

- N parties (entities/people splitting up stuff)

Try 1: A share is fair if it is worth $1/N$ of the total value.

Try 2: A share is fair to a specific party if it is worth at least $1/N$ of the total value according to that party's valuation of the stuff.

3. A starter example

| | |
|---|---|
| **What is being divided:** | 6 muffins<br>2 apple-walnut (A)<br>2 blueberry (B)<br>2 cheese & jalepeno (C) |
| **Cost:** | the package of 6 cost $12 ← total value |
| **Parties (or who is doing the splitting):** | Xavier (X), Yuri (Y), and Zariah (Z) |
| **Preferences:** | X likes all the flavors equally.<br>Y is allergic to all nuts so he can't eat the apple ones.<br>Z likes A twice as much as B or C |

(a) **Ignoring all preferences** and just using the cost of the package, what is the value of a muffin?

Total value = $12, so each muffin is worth $2

(b) **In a dollar amount**, what would be **the value of** a fair share in this case?

3 people, so a fair share is $12/3 = $4 worth of muffins.

(c) Fill out the table below indicating for each party (X,Y, or Z), the dollar amount **they** would assign to each muffin. The total value should always sum to $12. (!!)

| party | A | A | B | B | C | C | total for package |
|---|---|---|---|---|---|---|---|
| X | 2 | 2 | 2 | 2 | 2 | 2 | 12 |
| Y | 0 | 0 | 3 | 3 | 3 | 3 | 12 |
| Z | 2<br>3 | 2<br>3 | 1<br>1.5 | 1<br>1.5 | 1<br>1.5 | 1<br>1.5 | 8 ← No good!<br>12 Yay! |

12/8 = 1.5

4. **YOU** pick a division of the 6 muffins into three fair shares according to **Xavier's values.**

Lots of options.
Let's use AB / BC / CA

5. What is the value of ~~the~~ each share according to **Yuri's** values?

AB worth $3 CA worth $3 ← Not fair since not worth ≥ $4 to Yuri

BC worth $6 ← happy with this one! FAIR SHARE.

6. What is the value of the each share according to **Zariah's** values?

AB worth $4.50 ←

BC worth $3 ← Not fair to Zariah since worth less than $4

CA worth $4.50 ← happy with either of these! FAIR SHARE