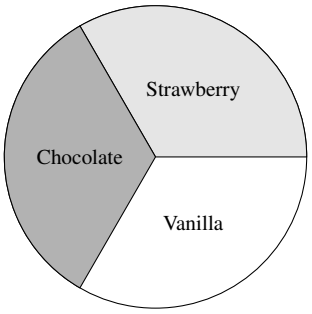
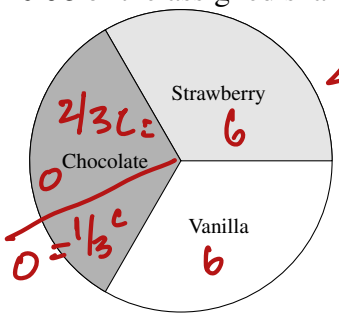
Goal: Review Divider-Chooser, Introduce Lone-Divider

1. Recall from the previous worksheet: Tom and Fred were given a cake worth \$12 that is equal parts strawberry, vanilla and chocolate, their respective values summarized in the chart.

| | vanilla | strawberry | chocolate |
|---|---|---|---|
| Tom | \$ 6 | \$ 6 | \$ 0 |
| Fred | \$ 2 | \$ 4 | \$ 6 |




(a) Divide the cake using Divider-Choose assuming Tom is the divider. Determine the **value** of the assigned share to each party.




Tom divides like this.

Tom's value: 6+0 / 6+0

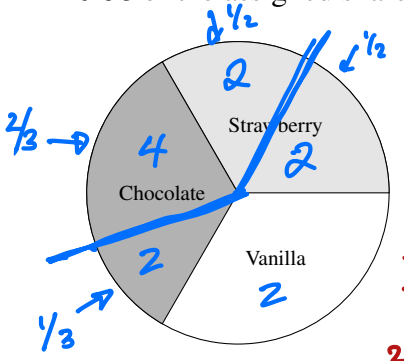
fair to Fred

Fred's value: straw/choc = $4 + \frac{2}{3}(6) = 4+4 = 8$

vanilla/chocolate = $\frac{1}{3}(6) + 2 = 3+2 = 5$

Fred chooses the strawberry + 2/3 chocolate piece worth \$8

(b) Divide the cake using Divider-Choose assuming Fred is the divider. Determine the **value** of the assigned share to each party.




Fred splits as follows:

2/3 chocolate + 1/2 strawberry / All vanilla, 1/2 S, 1/3 C

value = 6 value = 6

Tom values:

$\frac{2}{3}C + \frac{1}{2}S = \frac{2}{3}(0) + \frac{1}{2}(6) = 3$

$\frac{1}{3}C + 1V + \frac{1}{2}S = \frac{1}{3}(0) + 6 + \frac{1}{2}(6) = 9$ ← Fair share to Tom!

2. Is it better to be the Divider or the Chooser? Why?

Chooser – you can always pick an allocation that is worth at least a fair share, and often more than a fair share to you!

3. Lone-Divider Method (for $N$ people with $N \geq 3$).

   0. **Arbitrarily** pick a Divider.
   1. The Divider divides the items into $N$ shares of equal value to them: $s_1, s_2, \cdots, s_N$.
   2. The remaining parties **declare** or **bid** on which the shares, $s_1, s_2, \cdots, s_N$, they consider fair.
   3. i. **IF** the $N$ shares can be divided among the parties such that each gets a fair share, then do so.

      ii. **IF NOT**, then give the Divider a **non-contested piece**. Then restart Lone-Divider with $N-1$ parties and recombine the shares.

4. **Example 1** Suppose Patrick, Chris, and Travis are splitting a pile of football memorabilia estimated to be worth \$300. It has been split into 3 shares and their respective values are summarized in the table.

| | $s_1$ | $s_2$ | $s_3$ |
|---|---|---|---|
| Patrick | \$50 | \$150 | \$100 |
| Chris | \$70 | \$70 | \$160 |
| Travis | \$100 | \$100 | \$100 |

a) what is a fair share? 300/3 = \$100

(b) Circle or highlight each individual's **bid** (the shares they would consider to be fair).

(c) Determine which person was the Divider.

Travis! Splits are equal

(c) Determine the next steps of the Lone-Divider Method.

Can we allocate so that everyone gets a piece that is a fair share to them? Yes!

P gets $S_2$, C gets $S_3$, T gets $S_1$

5. **Example 2** Suppose Patrick, Chris, and Travis are splitting a pile of football memorabilia estimated to be worth \$300. It has been split into 3 shares and their respective values are summarized in the table.

| | ~~$s_1$~~ $t_1$ | ~~$s_2$~~ $t_2$ | ~~$s_3$~~ $t_3$ |
|---|---|---|---|
| Patrick | \$100 | \$100 | \$100 |
| Chris | \$90 | \$40 | \$170 |
| Travis | \$50 | \$90 | \$160 |

↑ (different)

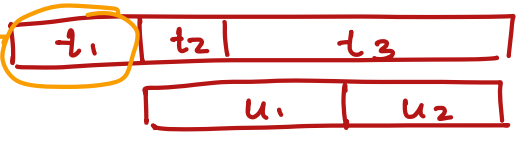
(a) Circle or highlight each individual's **bid** (the shares they would consider to be fair).

(b) Determine which person was the Divider.

(c) Determine the next steps of the Lone-Divider Method.

- Give patrick an uncontested share, say $t_1$
- Then Chris & Travis do divide-chooser on the entire collection $t_2 + t_3$ ← Choose a divider, divide up stuff, other chooses.

to P




Chris's new fair div.