1. Context for Weighted Voting

Different voters have different weights/ # votes
- US House - different states get different # votes
- Company - votes are weighted by # of shares

2. Notation and Terminology for Weighted Voting

- players

people/entities voting

$P_1, P_2, P_3, \ldots, P_N$

- weight

$w_1, w_2, \ldots, w_N$

total weight/# votes $= w_1 + w_2 + \cdots + w_N$

- quota

minimum weight to approve a motion/action/win

- coalition

group of players that vote together

- winning coalition

group of players whose weights sum to $\geq$ quota

- critical player

a player in a winning coalition whose removal causes the coalition to no longer be winning

3. Example: [25: 11, 11, 10, 8]

| player | $P_1$ | $P_2$ | $P_3$ | $P_4$ |
|---|---|---|---|---|
| weight | 11 | 11 | 10 | 8 |

quota = 25

total weight = 11+11+10+8 = 40

$\{P_1, P_2, P_3\}$ is a winning coalition since 11+11+10 = 32 > 25 and each player is critical (removal $\Rightarrow$ sum < q)

4. Reasonable Limits on the Quota

$q > \frac{1}{2}$ total ← you should need at least a majority

$q \leq$ total weight ← You should be able to have <u>some</u> winning coalition! (maybe everyone)

5. A Look at Power

(a) A Dictator [25: <u>25</u>, 10, 3, 2], total weight = 40

Someone who can make quota by themself

25 is quota, and $P_1$ has weight 25

(b) Having Veto Power [25: <u>21</u>, ~~10, 3, 2~~ 11 5 3], total weight = 40

Someone in every winning coalition / you can't make quota without them (always critical)

$11 + 5 + 3 = 19$ which isn't quota

So $P_2, P_3, P_4$ together aren't a winning coalition

so any winning coalition must include $P_1$

(c) A Dummy [25: 20, 18, <u>1</u>, <u>1</u>], total weight = 40

Someone whose vote isn't necessary for any winning coalition (never critical)

$P_1 + P_2 = 38$ wins

$P_1 + P_3 < q$

$P_1 + P_4 < q$ etc.

$P_1 + P_3 + P_4 < q$

So $P_3, P_4$ are not useful.