Goal: Understand how the Banzhaf Power Index is Calculated

1. **Review**: Answer questions about the weighted voting system below.

$$[67; 50, 30, 10, 5, 4, 1].$$

(a) How many players are there? 5

(b) What is the quota? 67

(c) What is the total vote? 100

$50 + 30 + 10 + 5 + 4 + 1 = 100$

(d) Is there a dictator? Explain.

No: no one can make quota by themself.

(e) Does any player have veto power? If so, find them and **justify** your conclusion.

quota is 60, and $30+10+5+4+1 = 50$. So players P_2 to P_5 can't make quota alone, so any winning coalition must include P_1. So P_1 has veto power.

(f) Are there any dummy players? If so, find them and **justify** your conclusion.

No winning coalition requires P_6

$P_1 P_3 P_4 P_6$ sums to $50+10+5+1 = 66 < q$

$P_1 P_3 P_5 P_6$ sums to $65 < q$

$P_1 P_3 P_4 P_5$ sums to 69

So adding P_6 doesn't make a difference.

All other winning coalitions (ex $P_1 P_2$) don't need P_6 either

2. Recall:

Coalition

group of voters that vote together

Winning Coalition

coalition whose votes sum to at least quota

Critical Player

player whose removal causes a coalition to go from winning to losing

3. Answer questions about the weighted voting system below.

$$[30; 25, 10, 5]$$

In general there are $2^N - 1$ coalitions (= # of "nonempty subsets") among N players

(a) List **ALL** coalitions.

Coalition	Sum
P_1	25
P_2	10
P_3	5
$P_1 P_2$	$25+10=35$
$P_1 P_3$	$25+5=30$
$P_2 P_3$	$10+5=15$
$P_1 P_2 P_3$	$25+10+5=35$

(b) List all **WINNING** coalitions.

only 3: $P_1 P_2$

$P_1 P_3$

$P_1 P_2 P_3$

(c) In each **winning** coalition, identify which players are critical.

$P_1 P_2$: if you remove either you drop below 30

$P_1 P_3$: if you remove either you drop below 30

$P_1 P_2 P_3$: only P_1 is critical, since $P_1 P_2$ is winning (don't need P_3) and $P_1 P_3$ is winning (don't need P_2)

(d) Calculate the Banzhaf Power index.

- ✓ i. Find all winning coalitions
- ✓ ii. Find all critical players
- ✓ iii. Underline critical players
- iv. Count total # underlines
- v. For each player, compute:

$$\frac{\#\text{ times that player is critical}}{\text{total \# underlines}}$$

$\underline{P_1 P_2}$

$\underline{P_1 P_3}$

$\underline{P_1 P_2 P_3}$

Total # underlines = 5

Banzhaf Power Index

Player	# times critical	$\frac{\#\text{ critical}}{\text{total \# underlines}}$
P_1	3	3/5
P_2	1	1/5
P_3	1	1/5
total # underlines →	5	

* observe! P_2 & P_3 have the same power even though P_2 has twice as many votes as P_3!