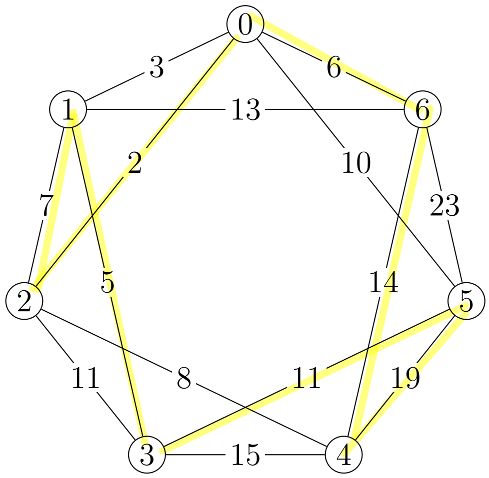
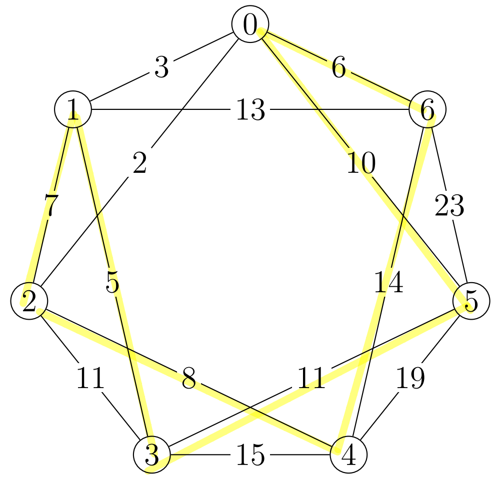
Hamiltonian Circuits and Paths (Day 2)

1. Recall the problem at the end of Worksheet 14.

   Use the Nearest Neighbor Algorithm starting at vertex 0 to find a Hamiltonian circuit. Highlight the circuit on the left-hand graph.





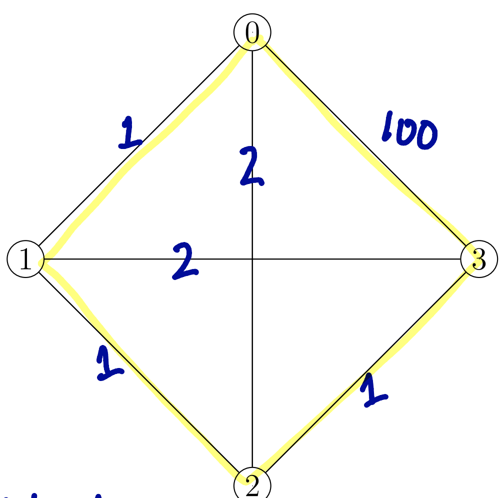
   List the vertices of the circuit in order: 0 2 1 3 5 4 6 0

   What is the weight of the circuit you found? 64

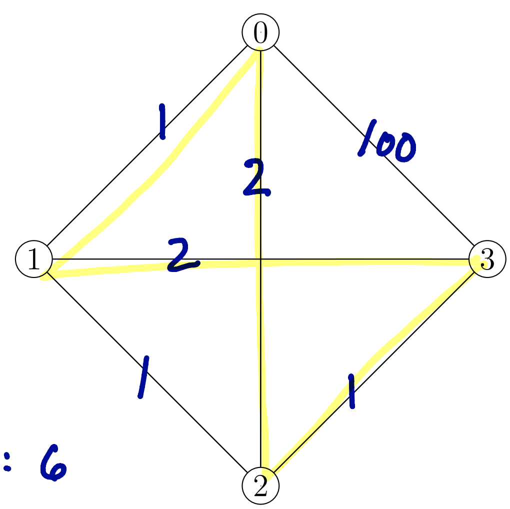
   The Hamiltonian circuit with smallest weight has weight 61. Can you find it? Draw it on the second graph. 0 6 4 2 1 3 5 0 weight 61

2. Add weights to the complete 4-vertex graph below such that NNA starting at vertex 0 gives the *highest* weight Hamiltonian circuit. (That is, show that NNA can give the worst possible answer!)

   On the left, show the Hamiltonian circuit obtained by starting NNA at 0. On the right, find the minimum weight Hamiltonian circuit. Calculate the weights of each.



weight : 6



Weight: 103

How do you *know* that circuit on the right is a minimum? Ad hoc... There is no other circuit without edge 0-3.

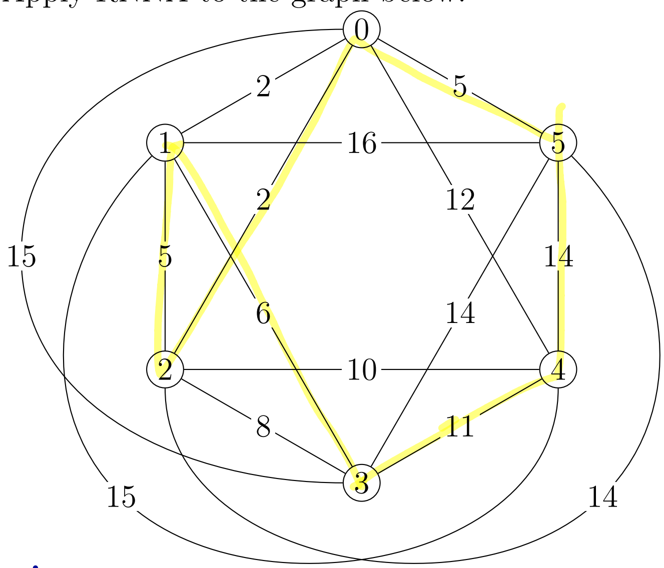
Will NNA ever give the circuit on the right? Maybe... if start @ 2 and goto 3 first.

3. Repeated Nearest Neighbor Algorithm:

Perform NNA starting at every vertex. Pick the circuit with smallest weight.

[How many times do we have to do NNA? Answer: #vertices!]

4. Apply RNNA to the graph below.



← I changed the #s here!

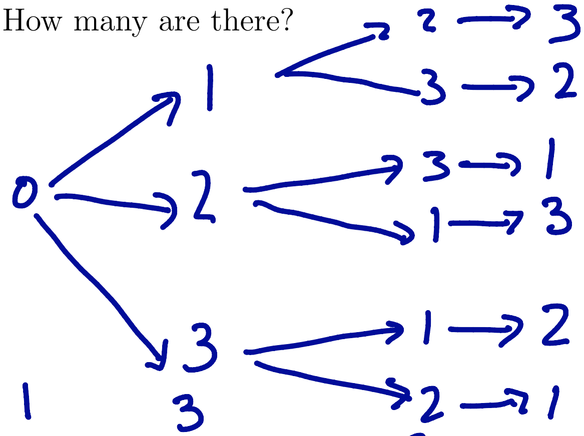
Starting vertex	Circuit	weight
0	0 1 2 3 4 5 0	2+5+8+11+14+5
1	1 0 2 3 4 5 1	2+2+8+11+14+16
2	2 0 1 3 4 5 2	2+2+6+11+14+14
3	3 1 0 2 4 5 3	6+2+2+10+14+14
4	4 2 0 1 3 5 4	10+2+2+6+14+14
5	5 0 1 2 3 4 5	5+2+5+8+11+14
	5 0 2 1	

5. How would be know if *any* of the circuits above are optimal? Check all??

6. Brute Force Algorithm : Check the weight of every cycle. Pick the smallest weight.

Can we do better?

7. Find a *systematic* way of listing all possible circuits on four vertices with vertex set 0,1,2,3. How many are there?



0.1.2.3.0
0 1 3 2 0
0 2 3 1 0
0 2 1 3 0
0 3 1 2 0
0 3 2 1 0

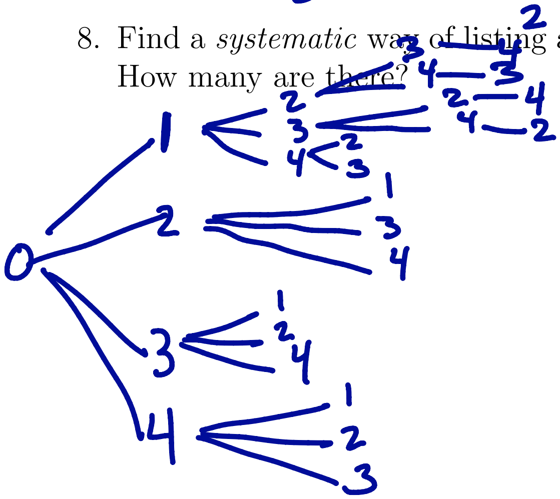
6 circuits

$3 \cdot 2 \cdot 1 = 6$

ok. ÷ by 2

So 3

8. Find a *systematic* way of listing all possible circuits on five vertices with vertex set 0,1,2,3,4. How many are there?



$4 \cdot 3 \cdot 2 \cdot 1 = 24$

ok $\frac{24}{2} = 12$

9. *Without* listing all possible circuits on six vertices with vertex set 0,1,2,3,4,5, how many are possible?

$\underline{5} \cdot 24 = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5!$

$\frac{5!}{2}$

10. How many different circuits on 100 vertices with vertex set 0,1,2,3,4,5,...,99?

$99! \approx 9.33 \times 10^{155}$

$\frac{99!}{2} \approx 4.66 \times 10^{155}$

$\boxed{\text{On } n \text{ vertices, there are } \frac{(n-1)!}{2} \text{ ham. circuits}}$

11. **Factorial Notation:**

$n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1$

$3! = 3\cdot 2\cdot 1$

$4! = 4\cdot 3\cdot 2\cdot 1$

$5! = 5\cdot 4\cdot 3\cdot 2\cdot 1$

Most calculators can't manage more than $\approx 60!$

12. **Scientific Notation:**

$2,347,289 \approx 2.34\times 10^{6}$

add 1

$10! = 3.6\times 10^{6}$

$11! = 3.99\times 10^{7}$

double input

$20! = 2.43\times 10^{18}$ ← more than # grains of sand

$40! = 8.15\times 10^{47}$ ← # atoms on earth
close

13. How do you calculate $n!$ for very large n-values?

Wolfram Alpha or oth