MATH F113X: Euler Circuits and Paths

The goals are to understand:

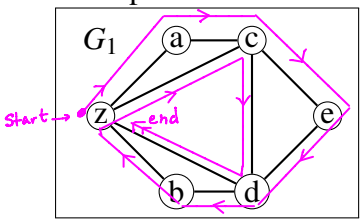
- the definition of an Euler circuit and an Euler path.
- applications of Euler circuits and paths.
- how to use vertex degrees to determine if Euler circuits and/or paths exist.
- how to find and describe Euler circuits and/or paths.

1. Definitions

   (a) **(Euler circuit)** is a circuit that includes every edge of the graph. (Recall that the circuit must start and stop at the same vertex and it cannot use an edge twice)

   (b) **(Euler path)** is a path that includes every vertex. (Recall that every circuit is a path but not every path is a circuit.)

2. Examples



Euler circuit (and path)

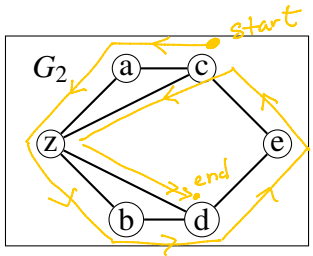
z a c e d b z c d z

OR

a c e d b z c d z a

OR

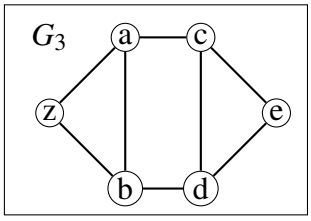
c e d b z c d z a c ...



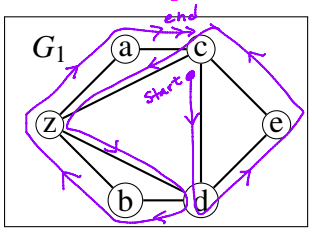
Euler path (NOT a circuit)

c a z b d e c z d

This graph has NO Euler circuit.

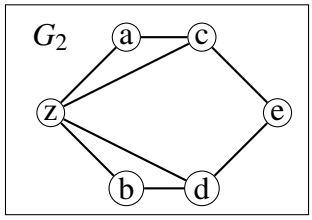


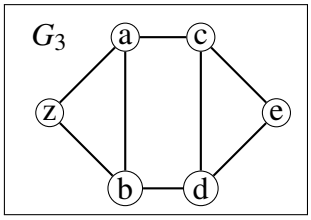
This graph has no Euler path or circuit.



euler circuit

c d e c z d b z a c





3. When are we guaranteed there is an Euler circuit and why?

If the graph is connected and all vertices have even degree, then the graph always has an Euler circuit.
With even degrees, every time the path enters a vertex, there is always some edge available to leave.

4. When are we guaranteed there is an Euler path and why? ← and NOT a circuit

If the graph is connected and has exactly two vertices of odd degree (so all others have even degree), then there is always an Euler path between the vertices of odd degree.
The path will leave one vertex of odd degree and not return. It will enter the other vertex of odd degree and not leave.

5. When are we guaranteed there is not an Euler circuit and there is not an Euler path and why?

If a graph has more than two vertices of odd degree, then it does not have an Euler path or circuit.

6. If there is an Euler circuit, how can you find it?

You can pick an arbitrary vertex, follow edges until returning to the start. Append circuits until all edges are used.

7. If there is an Euler path, how can you find it?

Start at one odd vertex, follow edges until achieving a path to the second odd vertex. Append circuits until all edges are used.

8. Are Euler circuits or Euler paths **unique**?

No. See the examples in #2 on prev. page.