MATH F113X: Sortest Edges/Chepaest Link Algorithm for Hamiltonian cycles

The Sorted Edges / Cheapest Link Algorithm

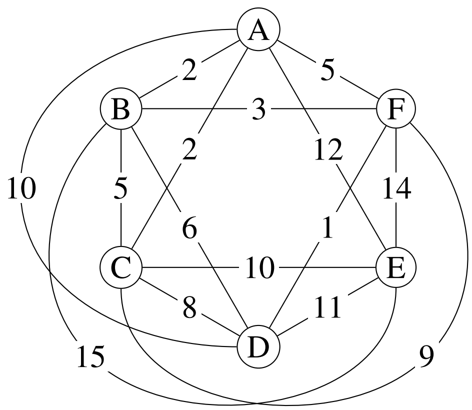
input: a graph with distances (weights) on the edges

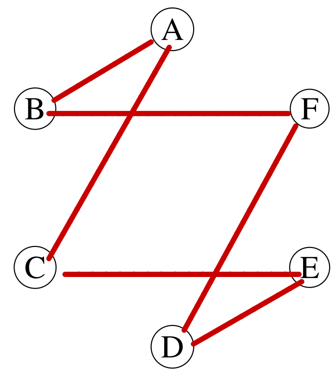
output: a Hamiltonian circuit (or the algorithm fails)

strategy: Add the next cheapest edge to your circuit unless it closes the circuit too soon or creates a degree 3 vertex. Break ties by choosing the alphabetically smallest edge.

Note that you may have disconnected edges during the process of running the application.

For convenience, I have listed the edges of the graph in sorted order for you. Draw in the edges as you add them on the empty graph.





Sorted edges	weight	used? ~~(or why not)~~
FD	1	✓
AB	2	✓
AC	2	✓
BF	3	✓
~~AF~~	~~5~~	
~~BC~~	~~5~~	
~~BD~~	~~6~~	
~~CD~~	~~8~~	
~~CF~~	~~9~~	
~~AD~~	~~10~~	
CE	10	✓
DE	11	✓
AE	12	
EF	14	
BE	15	

B would have degree 3!

Would close the cycle too early.

List the vertices of the Hamiltonian circuit, starting at vertex A.

A B F D E C A

What was the weight of the circuit you found? $1+2+2+3+10+11=29$

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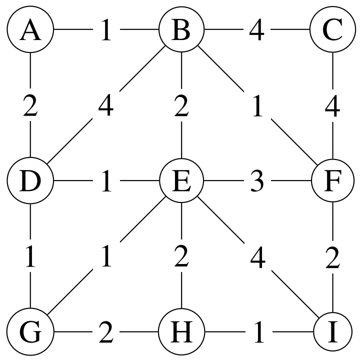
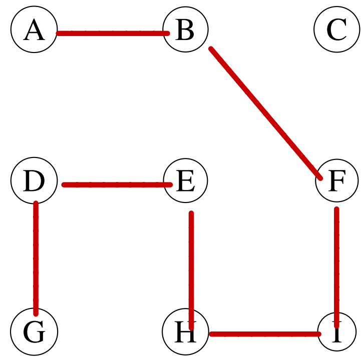
What happens if you apply Sorted Edges/Cheapest Link to the following graph?





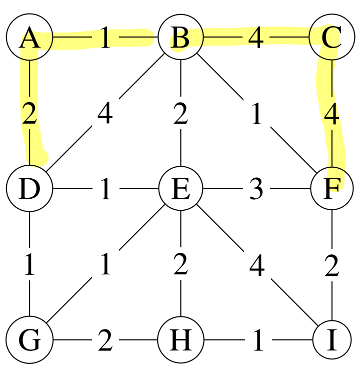
No cycle is obtained.

Sorted edges	weight	used? (~~or why not~~)
• AB	1	✓
• BF	1	✓
• DE	1	✓
• DG	1	✓
• ~~EG~~	~~1~~	
• HI	1	✓
• ~~AD~~	~~2~~	
• ~~BE~~	~~2~~	

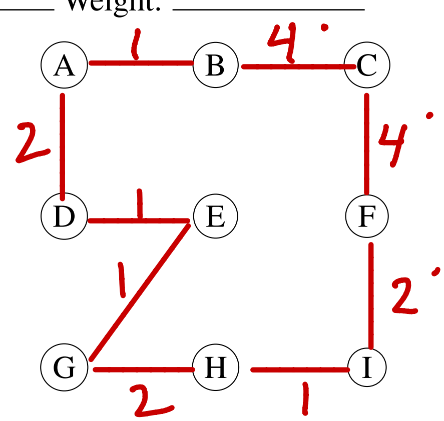
Sorted edges	weight	used? (~~or why not~~)
• EH	2	✓
• FI	2	✓
• ~~GH~~	~~2~~	
• ~~BC~~	~~4~~	
• ~~BD~~	~~4~~	
• ~~CF~~	~~4~~	
• ~~EI~~	~~4~~	

There is a Hamiltonian circuit on this graph. What is the smallest-weight Hamiltonian circuit you can find?

Circuit: ABCFIHGEDA Weight: 18



• These edges have to be there



One strategy for having the algorithm not fail is to turn your graph into a complete graph, and assign very expensive weights (say, 100000) to the "non-edges". For example, there is no edge between G and A, so you could add edge AG and give it a weight of 10000.

The complete graph on 9 vertices has 36 edges.

1. How many edges would you need to add to turn this graph into a complete graph? $36-15=21$
2. In this new graph, how many possible Hamiltonian circuits are there? $\frac{9!}{2}$ or $9!$

(depends on symmetry)