MATH F113X: Eulerization

Goals: how to Eulerize a graph; why you would Eulerize a graph; how to put Dijkstra's algorithm together with Euler circuits (worksheet)

1. Given a graph, when can you find:
   (a) An Euler circuit? All vertices have even degree.

   (b) An Euler path? Zero or two vertices have odd degree. All others have even degree

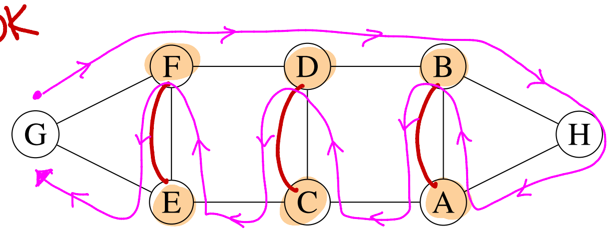
   (c) Neither? More than two vertices have even degree

2. Recall problem 5 from Worksheet 12:

   Double some of the edges so that every vertex is even degree. Using your additional edges, find an Euler circuit.

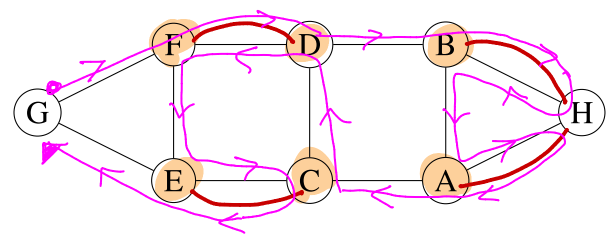
OK

added 3 edges

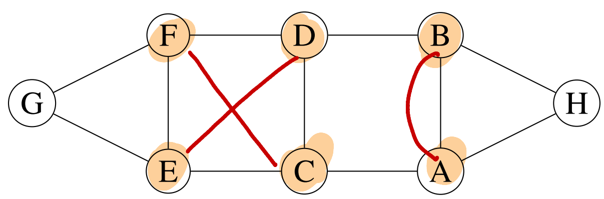


added 4 edges

OK

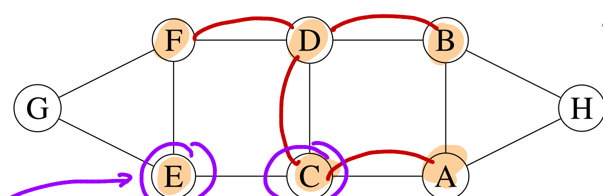


bad



No making up edges.

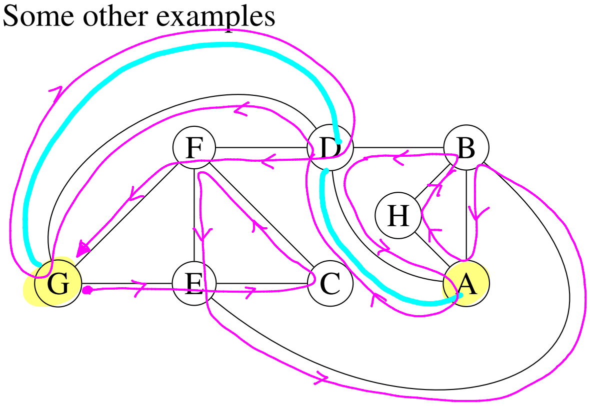
bad

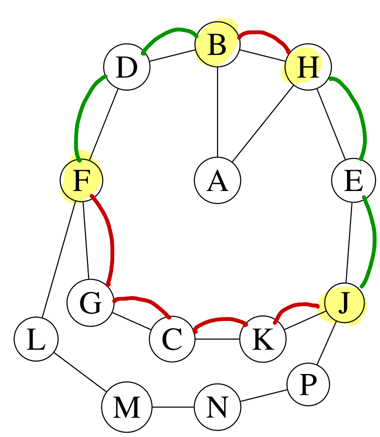


We need ALL vertices to be even.

still odd.

3. Some other examples





- option 1 add 5 edges
- option 2 add 4 edges

Lesson: Find the optimal (best) Eulerization can be tricky.

4. **Definition:** To **eulerize** a graph *G* means to duplicate existing edges in the graph G so that all vertices have even degree.

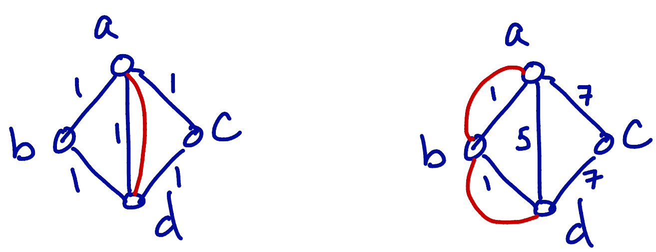
FYI: One option is to duplicate EVERY edge.

Thus, it is always possible to Eulerize a (connected) graph.

5. **Definition:** An **optimal eulerization** means

- (unweighted graph) to duplicate the fewest number of edges.
- (weighted graph) the total weight of the duplicated edges is smallest possible.

Ex



6. Under what conditions do you think it is *easy* to obtain an optimal eulerization?

0 odd-degree vertices

- The graph has all even degree and there's nothing to do!

2 odd-degree vertices

- If the graph has exactly two vertices of odd degree, (say A and B), then the shortest path from A to B is optimal. (Use Dijkstra's Algorithm!)
- In all other cases, it can be hard.