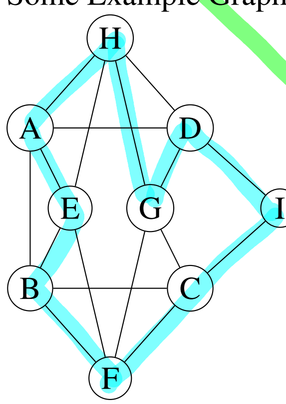
MATH F113X: Introduction to Hamiltonian Circuits and Paths

Terminology: Hamiltonian Path, Hamiltonian Circuit

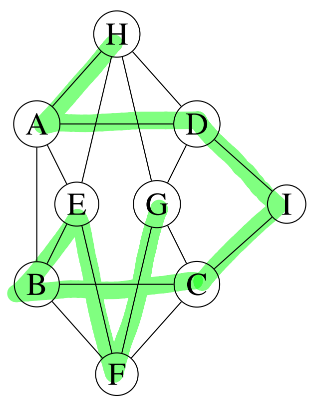
1. A **Hamiltonian circuit** (sometimes called Hamiltonian Cycle) is a circuit that includes every vertex exactly one time

2. A **Hamiltonian path** is a path that contains every vertex exactly one time.

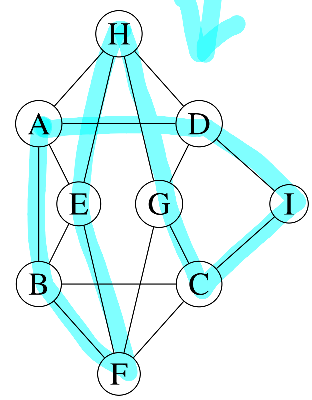
3. Some Example Graphs



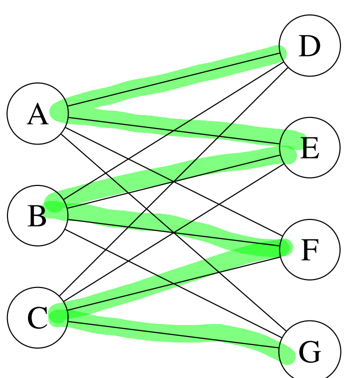
AHGDICFBEA
Hamitonian circuit



HADICBEFG
Hamitonian path.

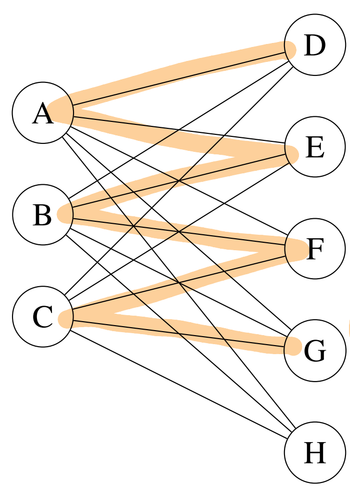


not unique



Hamiltonian path.

No Hamiltonian circuit is possible.



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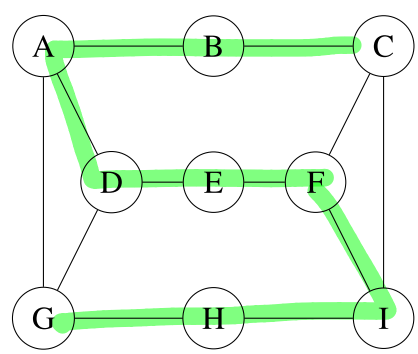
No Hamiltonian path. No Hamiltonian circuit.

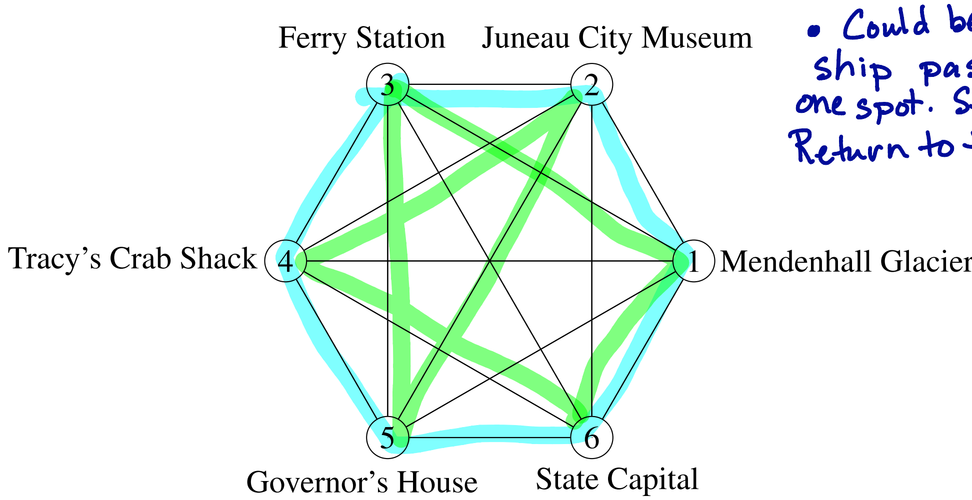


Hamitonian path.
No Hamiltonian circuit.

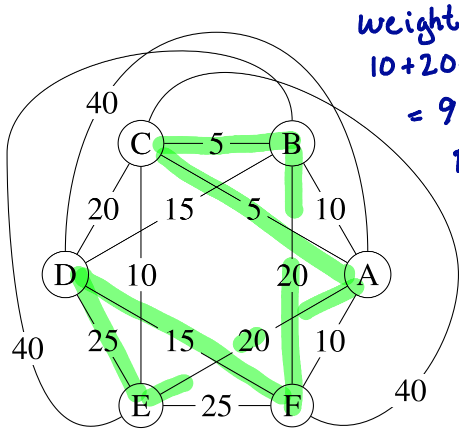
5. What would a Hamiltonian circuit in the graph below represent?

• Many different hamitonian circuits

• Could be a tour for cruise ship passengers. Meet at one spot. See all the sights. Return to that spot.

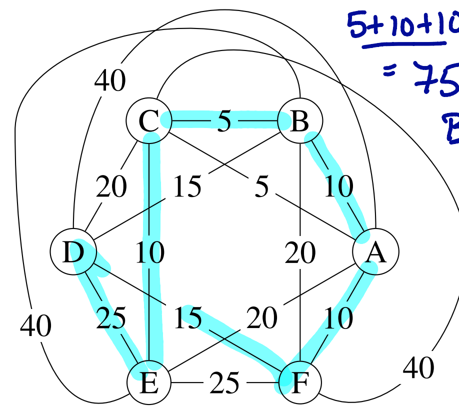


6. Why might you want to find a Hamiltonian circuit of smallest weight? How might you do that?



weight:
10+20+20+40
= 90

Best possible?



weight:
5+10+10+15+25+10
= 75

Best possible?

Why? Smallest weight might represent the cheapest route or the fastest route.

How? Uh... try to use edges with low weight??

7. Comments on a concrete application and counting.

UPS/FedEx driver. Typically has 50-120 stops/day. Starts and ends @ distribution center. If you could regularly find an efficient route that was 1 hour faster, would it matter?

(≈ 100,000 drivers/day ; Saving 4,166 hrs/day)

How many different routes are possible?

Suppose our UPS driver has 50 stops (not including the depot.)

depot	1st stop	2nd stop	3rd stop	...	49th stop	50th stop	back to depot
start	50 choices	49 choices	48 choices		2 choices	1 choice	
no choice							

$$\# \text{ routes} = 50 \cdot 49 \cdot 48 \cdot \ldots \cdot 3 \cdot 2 \cdot 1 = 50!$$

$$> 3 \times 10^{64}$$

Even supposing 10^{16} checks/sec, checking all possibilities takes 3×10^{48} seconds

$$\approx 9.5 \times 10^{40} \text{ years}$$

For context, the universe is $\approx 1.4 \times 10^{10}$ years.