MATH F113X: Sortest Edges/Cheapest Link Algorithm for Hamiltonian cycles

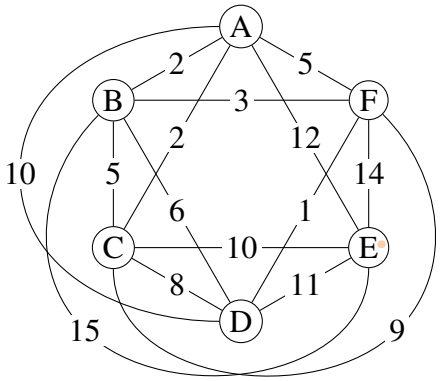
The Sorted Edges / Cheapest Link Algorithm

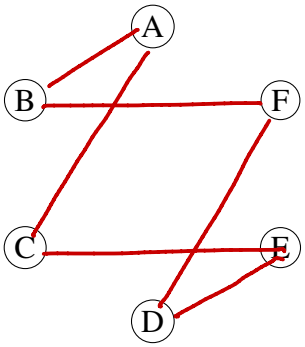
Steps: Add the next cheapest edge to your circuit **unless**

1. it closes the circuit too soon, or
2. creates a degree 3 vertex.

Break ties by choosing the alphabetically smallest edge.

Apply the Sorted Edges Algorithm to find a Hamiltonian circuit. Draw in the edges, labeled with their weight, as you add them on the empty graph.





Sorted edges	weight	used? (or why not)
FD	1	✓
AB	2	✓
AC	2	✓
BF	3	✓
AF	5	✗ deg 3 at A
BC	5	✗ deg 3 at B
BD	6	✗ deg 3 at B
CD	8	✗ closes early
CF	9	✓ deg 3 at F
AD	10	✗ deg 3 at A
CE	10	✓
DE	11	✓
AE	12	
EF	14	
BE	15	

List the vertices of the Hamiltonian circuit, starting at vertex A.

A B F D E C A

Total weight of the circuit?

11+10+3+2+2+1 = 29

Do you think the circuit we obtained in the best possible?

maybe?

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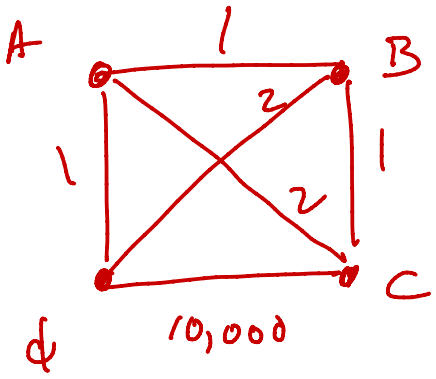
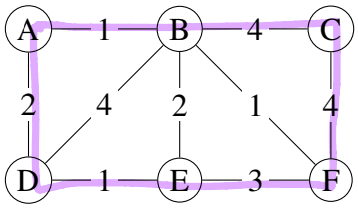
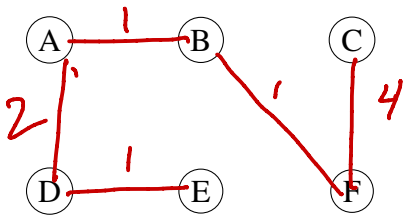
Can you construct a graph such that the Sorted Edges Algorithm will never result in a Hamiltonian circuit of smallest weight? What does this tell us about the Sorted Edges Algorithm?



What happens if you apply Sorted Edges/Cheapest Link to the following graph?





Sorted edges	weight	used? (or why not)
AB	1	✓
BF	1	✓
DE	1	✓
AD	2	✓
BE	2	X closes

Sorted edges	weight	used? (or why not)
EF	3	X closes
BC	4	X deg 3 @ B
BD	4	X deg 3 @ B
CF	4	✓

What is the problem here?

There are not enough edges to close the circuit.
The graph is not complete!

There is a Hamiltonian circuit on this graph. What is the smallest-weight Hamiltonian circuit you can find?

- ABCD FEDA Weight 15

Circuit: _____ Weight: _____