1. The context of Fair Division

- People/entities are dividing some stuff
  - Company dividing assets - roommates dividing chores/stuff
  - heirs dividing estate
- Different people value things differently
- Assumptions: Individuals can assign some value to stuff and make choices based on that value.
- Bad: An individual doesn't value X, they just don't want someone else to get it.

2. The definition of **a fair share**:

Suppose there are N parties splitting the stuff.

Try 1 → Then a fair share is $\frac{1}{N}$ of the total value.

Refine to Try 2 A fair share for a specific party is one with value at least $\frac{1}{N}$ of the total value... according to the value of the specific party.

3. Divider-Chooser in a Nutshell Two parties.

- Assign one party to be the Divider; the other is the Chooser
- The Divider splits stuff into two equal shares: S_1 and S_2
- The Chooser picks their preferred share.
- The Divider gets the remaining share.

* Using Algebra
x = value of B or C

Example

What is being divided: 6 muffins
2 apple-walnut (A)
2 blueberry (B)
2 cheese & jalepeno (C)

2x = value of A.

Now $12 = 2(2x) + 4(x) = 8x$.

So $x = \frac{12}{8} = 1.5$ ✓

Cost: the package of 6 cost $12

Parties (who is involved): Yuri (Y), and Zariah (Z)

Preferences: Y likes all the flavors equally.
Z likes A twice as much as B or C

(a) **Ignoring all preferences**, what is the value of a muffin? $12/6 = $2

(b) **In a dollar amount**, what would be **the value of** a fair share in this case? $\frac{\$12}{2} = \$6$

(c) Fill out the table below indicating for each party (X,Y, or Z), the dollar amount **they** would assign to each muffin. The total value should always sum to $12. (!!)

party	A	A	B	B	C	C	total value
Y	2	2	2	2	2	2	6·2 = 12 ✓
Z	2	2	1	1	1	1	← 1st try only $8
* Z	3	3	1.50	1.50	1.50	1.50	2·3 + 4·(1.50) = 12 ✓

(d) Complete Divider-Chooser with Yuri as the divider and Zariah as the chooser.

	Shares	Yuri's Value	Zariah's Value
S_1:	AAB	3·$2 = $6	3+3+1.50 = $7.50
S_2:	BCC	3·$2 = $6	3(1.50) = $4.50

Z choose S_1 worth $7.50

Y gets S_2 worth $6.00

Both receive shares worth at least $6.00

(e) Complete Divider-Chooser with Zariah as the divider and Yuri as the chooser.

Shares	Z's value	Y's value
S_1: AA	3+3 = 6	$4
S_2: BBCC	4(1.50) = 6	$8

Y chooses S_2 worth $8

Z gets S_1 worth $6.