Some Solutions

1. In earlier notes, we found seven possible coalitions with players P_1, P_2, and P_3.

   (a) List them again below.

   P_1
   P_2
   P_3
   P_1, P_2
   P_1, P_3
   P_2, P_3
   P_1, P_2, P_3

   total number: 7

   (b) Suppose the system has a fourth player, P_4. Determine how many coalitions in this case. Try to answer the question without actually listing all of them.

   Observation 1: All of the coalitions above are also coalitions with players P_1, P_2, P_4,and P_4. So we get the 7 above.
   Observation 2: Every time we add P_4 to a coalition above, we get another one. For example, adding P_4 to the first three in the list gives:

   P_1, P_4
   P_2, P_4
   P_3, P_4

   Observation 3: We missed out coalition P_4

   total number: $7+7+1 = 15$

   (c) What if there is a fifth player, P_5?

   guess: $15+15+1 = 31$

   (d) Make a conjecture about how many coalitions are possible with n players, $P_1, P_2, P_3, \cdots P_n$. How would you argue that your count is correct?

   What is the pattern you see?
   $7 = 8-1 = 2^3 - 1$
   $15 = 16-1 = 2^4 - 1$
   $31 = 32-1 = 2^5 - 1$
   guess: With n players there are $2^n - 1$ different coalitions.

(e) What does this suggest about the mechanics of calculating the Banzhaf Power Index for a weighted voting system with a lot of players?

2^n grows really really fast.
While $2^5 = 32$, $2^{10} = 1024$, and $2^{20} > 1,000,000$.
Listing **all** coalitions is, in general, not practical.

2. What is the Banzhaf Power Index

(a) when there is a dictator

dictator has 100%, all others have 0%

(b) for a dummy player

always has 0%

(c) if all players have an equal number of votes

All players have equal power. Might has well have 1 player 1vote.

(d) if player P_1 has double the number of votes as player P_2?

All you can conclude is that P_2 will not have **more** power than P_1. But beyond that there isn't enough information. We have seen examples in which such players have equal power and we can construct others where P_1 is a dictator: $[10 : 10, 5, 2]$