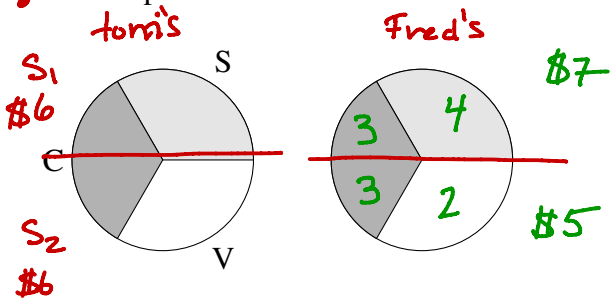
Goal: Review Divider-Chooser, motivate and introduce Lone-Divider

1. Recall from the previous worksheet: Tom and Fred were given a cake worth \$12 that is equal parts strawberry, vanilla and chocolate. Their respective values summarized in the chart.

	vanilla	strawberry	chocolate
Tom	\$ 6	\$ 6	\$ 0
Fred	\$ 2	\$ 4	\$ 6



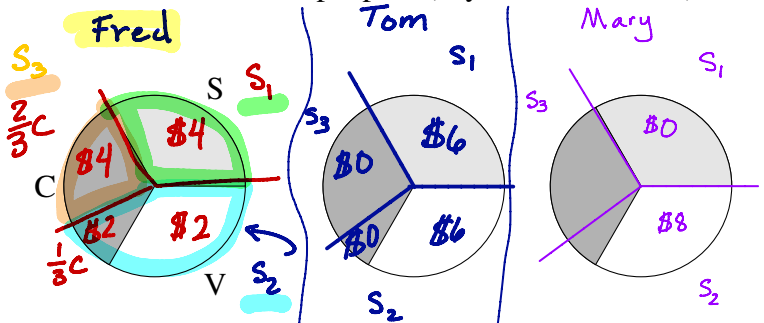
(a) Divide the cake using Divider-Chooser assuming Tom is the divider. Determine the **value** of the assigned share to each party. Use the pictures of the cake (above) to draw the outcome.)

outcome:

person	share	value
Tom	$S_2 = \frac{1}{2}C + V$	\$6
Fred	$S_2 = \frac{1}{2}C + S$	\$7

(b) What happens if try to use Divider-Chooser with **three** people? (Try Fred as divider.)

	V	S	C
Tom	\$ 6	\$ 6	\$ 0
Fred	\$ 2	\$ 4	\$ 6
Mary	\$8	\$ 0	\$ 4



Fair Share? $\frac{\$12}{3} = \$4$

$\frac{\$2}{\$6} = \frac{1}{3}$

S_1 or S_2 is fair

S_3 value: $\frac{2}{3} \cdot 4 = \frac{8}{3} \approx \$2.67$

S_2 value: $8 + \frac{1}{3}4 \approx \$9.33$

Scenario 1

- Fred divides
- Tom chooses S_1
- Mary chooses S_2
- Fred gets S_3

	value
	\$6
	\$9.33
	\$4

Scenario 2

- Fred divides
- Tom chooses S_2
- Mary chooses ... No remaining share is worth at least \$4!

Q| Why does D.C. always work w/ two people but does not always work w/ three (or more)?

2. Why cake?

↳ and not... a car... or pieces of furniture?

Answer: We need to be able to split them into smallish pieces.

3. Lone-Divider Method (for N people with $N \geq 3$).
   0. **Arbitrarily** pick a Divider.
   1. Divider divides the items into N shares of equal value to them: $s_1, s_2, \cdots, s_N$.
   2. Remaining parties **declare** or **bid** on shares, $s_1, s_2, \cdots, s_N$, they consider fair.
   3. i. **IF** the N shares can be divided such that each all get a fair share, then do so.
      ii. **IF NOT**, the Divider gets a **non-contested piece**. Remaining shares are recombined. Lone-Divider is restarted with $N-1$ parties.
      iii. If only 2 parties remain, use Divider-Chooser.

4. **Example 1** Suppose Patrick, Chris, and Travis are splitting a pile of football memorabilia estimated to be worth \$300. It has been split into 3 shares and their respective values are summarized in the table.

   (a) What is a fair share? $\frac{300}{3} = \$100$

	s_1	s_2	s_3
Patrick	\$50	\$150	\$100
Chris	\$70	\$70	\$160
Travis	\$100	\$100	\$100

   (b) Circle or highlight each individual's **bid** (the shares they would consider to be fair).

   (c) Determine which person was the Divider.
   Travis. All shares have equal value.

   (d) Determine the next steps of the Lone-Divider Method.

person	bid	assignment	assigned value	fair?
Patrick	s_2, s_3	s_2	\$150	✓
Chris	s_3	s_3	\$160	✓
Travis	s_1, s_2, s_3	s_1	\$100	✓

5. **Example 2** Suppose Patrick, Chris, and Travis are splitting a pile of football memorabilia estimated to be worth \$300. It has been split into 3 (different) shares and their respective values are summarized in the table.

	t_1	t_2	t_3
Patrick	\$100	\$100	\$100
Chris	\$90	\$40	\$170
Travis	\$50	\$90	\$160

   (a) Circle or highlight each individual's **bid** (the shares they would consider to be fair).

   (b) Determine which person was the Divider.
   Patrick

   (c) Determine the next steps of the Lone-Divider Method.

- Chris and Travis only view t_3 as fair. It is contested.
- Patrick gets t_1 (or t_2... it wouldn't matter...)
- Shares t_2 and t_3 are recombined. Then Chris and Travis proceed with Divider-Chooser.