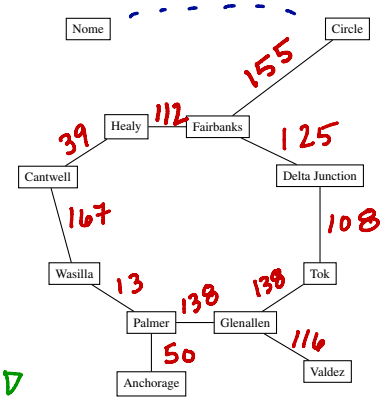
MATH F113X: Graph Theory Intro

1. **Example:** Some cities in Alaska

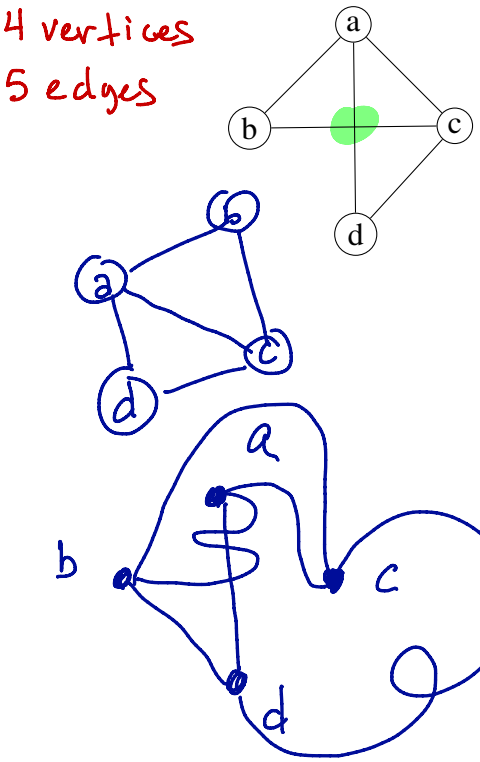


No edge here!

- Simple compared w/ atlas, and holds much of the same information. Can I get to Valdez from Circle by car? To Nome?
- Could make it even more robust by adding distances.
- If road from DJ to Tok is closed, how far from Fairbanks to Valdez?
- How many road closures would separate Fairbanks and Valdez?

Idea: Dots + Lines can summarize and simplify information we care about!

2. **vertex** (plural: vertices), **edge, graph**; ways to represent graphs



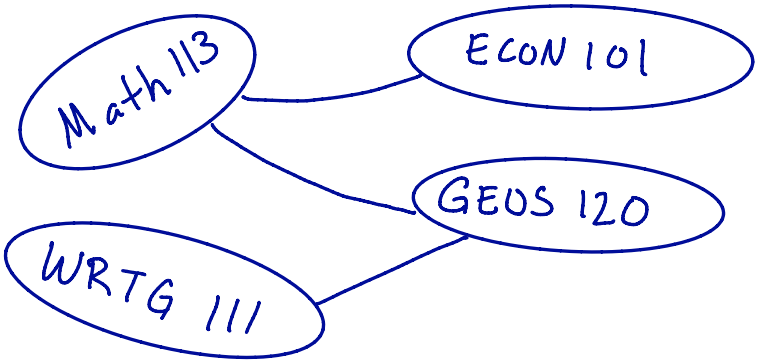
4 vertices
5 edges

- Many ways to draw a graph.
- where edges "cross" may not be a vertex
- Edges don't have to have weights and don't have to be straight.
- Say "vertices a and b are adjacent" "vertices b and d are not adjacent" "ab is an edge" or "bd is not an edge."

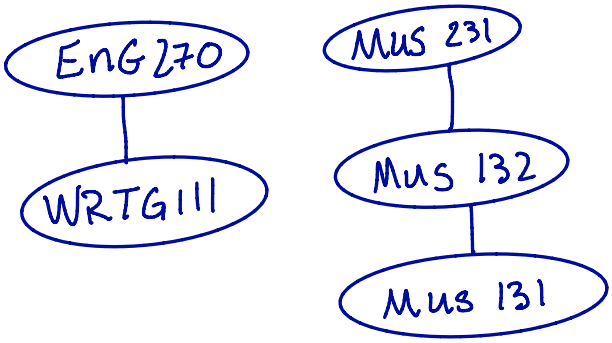
← weird but OK

Your ideas?

3. **Example:** Vertices are **classes**. What might edges represent?

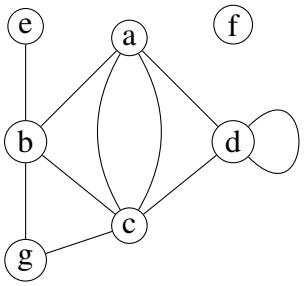


- Vertices: classes @ UAF
- edge between two classes if there is at least one student taking both classes.



- Vertices: classes
- edges: An edge between two classes if one is a prerequisite for another.

4. **Degree** of a vertex



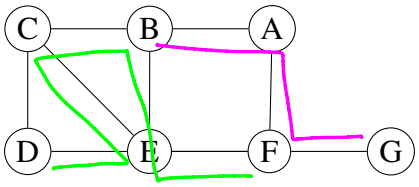
7 vertices
10 edges

is the number of edges incident to it

vertex	degree
a	4
b	4
f	0
d	4
e	1

← loops count twice!

5. **Path** in a graph

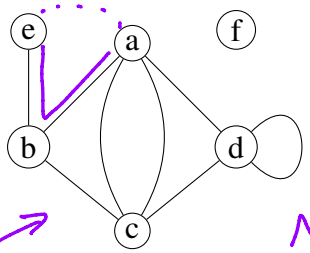


a sequence of vertices and edges with no repeated edges.

Examples
- GFAB
- DECBEF

Bad: FGA ← No edge GA!!
GFEFG ← can't go back over an edge!

6. A graph is **connected** if... there is a path between every pair of vertices.

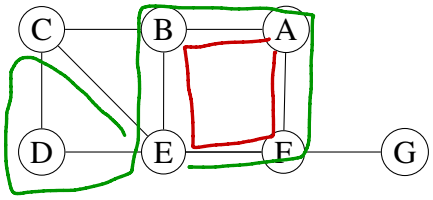


← This graph is not connected b/c no path from a to f.
The graph in #5 is connected.

Note: It is important to distinguish between edges and paths.

In this graph, there is no ae edge, but a and e are connected because there is a path from a to e.

7. A **circuit** in a graph... is a path that starts and ends at the same vertex



Examples:
- ABEFA
- EFABEDCE

Bad:
- GFA ← clearly no circuit using G!!
- DCEDCE ← can't reuse an edge.