Name: Solutions score: 10 / 10

There are 10 points possible on this quiz. No aids (book, notes, etc.) are permitted. You may use a non-programmable calculator. **Show all work and supporting calculations for full credit. Explain how you get your answers.**

1. (4 pts. total – 1 pt. each) For the weighted voting system $[11;7,5,2,2]$,

   (a) Why is $\{P_1,P_2,P_4\}$ a winning coalition?

   $w_1+w_2+w_4 = 7+5+2 = 14 \geq 11 = q$

   (b) Is P_2 a critical member of $\{P_1,P_2,P_4\}$? Show how you decided.

   $w_1+w_4 = 7+2 = 9 < 11$, so $\{P_1,P_4\}$ is not winning. Therefore P_2 is a critical member.

   (c) Are there any dictators for this voting system? If so, who? Explain your reasoning.

   There are no dictators, since no player weight is $\geq q = 11$

   (d) Which players, if any, have veto power? Explain your reasoning.

   P_1 has veto power, since $\{P_2,P_3,P_4\}$ has weight $5+2+2 = 9 < 11$ & is not winning. P_2, P_3, P_4 do not have veto power.

2. (3 pts.) For the weighted voting system $[14:9,6,6,2]$ there are 7 winning coalitions, listed here, with critical players underlined.

   $\{\underline{P_1},\underline{P_2}\}$ $\{\underline{P_1},P_2,P_3\}$ $\{\underline{P_1},\underline{P_3},P_4\}$ $\{P_1,P_2,P_3,P_4\}$

   $\{\underline{P_1},\underline{P_3}\}$ $\{\underline{P_1},\underline{P_2},P_4\}$ $\{\underline{P_2},\underline{P_3},\underline{P_4}\}$

   12 underscores total

   Using this information, compute the Banzhaf Power Index of each player. You may leave your answers as fractions.

   P_1: $\dfrac{\#\text{ of times } P_1 \text{ is critical}}{\#\text{ of critical players}} = \dfrac{5}{12}$

   P_2: $\hookrightarrow \dfrac{3}{12} = \dfrac{1}{4}$

   P_3: $\dfrac{3}{12} = \dfrac{1}{4}$

   P_4: $\dfrac{1}{12}$

3. (3 pts. total – 1 pt. each) A small business uses a weighted voting scheme $[q : 4, 2, 2, 1]$ for its four owners, with weights in proportion to the amount each invested.

   (a) One player suggests the quota be 2/3 of the total weight. What is the total weight, and what would be the quota?

   total weight = $4+2+2+1 = 9$

   quota = $\frac{2}{3} \cdot 9 = 6$

   (b) P_4 responds, "But that quota makes me a dummy!" Explain what this means. (You do not need to show that statement is correct, although it is.)

   P_4 is never critical; if P_4 drops out of any winning coalition, it still loses

   (c) If instead the group sets the quota to be 5, so the weighted system is $[5; 4, 2, 2, 1]$, then P_4 is not a dummy. Give a coalition that shows this, and explain why.

   $\{P_1, P_4\}$ has weight $4+1=5=q$ so it is winning. But, if P_4 drops out $\{P_1\}$ has weight $4 < q$ so is not winning.