## Worksheet 5: Weighted Voting and Banzhaf Power index

**Group Names:** _Solutions._

In each of the following weighted voting scenarios,

(a) Calculate the Banzhaf Power distribution.

(b) Explain whether you think the weighted voting system is fair using your results from the Banzhaf Power Distribution.

**Scenario 1** Three friends decided to start a small side business in their free time. In the beginning, they were all spending equal amount of time on the business. As the business grew, they quickly realized that some people were spending more time tending to business matters than others: Johnny was working 11 hours per week, Sally was working 8 hours per week, and Ramón was working 4 hours per week. Wanting to create fairness in the decision-making processes, they decided to base the weight of their votes in decision making off of the number of hours each one was averaging per week. The weighted voting system is represented by [12: 11, 8, 4].

Let's list all possible coalitions. Need 12 to win

4 winning coalitions.

1. $\{P_1\}$  $11 < 12$  X

2. $\{P_2\}$  $8 < 12$  X

3. $\{P_3\}$  $4 < 12$  X

4. $\{P_1, P_2\}$  $11 + 8 = 19 > 12$ ✓     ← all critical   $19 - 8 = 11 < 12$
                                                          $19 - 11 = 8 < 12$

5. $\{P_1, P_3\}$  $11 + 4 = 15 > 12$ ✓     ← all critical   $15 - 11 = 4 < 12$
                                                          $15 - 4 < 11 < 12$

6. $\{P_2, P_3\}$  $8 + 4 = 12$ ✓     ← all critical

7. $\{P_1, P_2, P_3\}$  $11 + 8 + 4 = 23 > 12$ ✓     None critical!
                                                          $23 - 4 = 19 > 12$
                                                          $23 - 8 = 15 > 12$
                                                          $23 - 11 = 12 = 12$

Critical votes:     Power

$P_1$: 2          $P_1$: $2/6 = 1/3$

$P_2$: 2          $P_2$: $2/6 = 1/3$

$P_3$: 2          $P_3$: $2/6 = 1/3$

total = 6

Everyone has the _same_ power despite very different hours worked. This doesn't seem very fair to me!

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**Scenario 2:** Dunder Mifflin needs a new manager. Instead of the same old hiring team, Robert
California decided to construct a team based on seniority. The number of votes each member
of the team got was also based on how long they had been a member of the Dunder Mifflin
family. Stanley Hudson joined Dunder Mifflin in 1985 giving him 30 points, Creed Bratton
joined in 1991 giving him 24 points, Phyllis Vance joined in 2001 giving her 14 points, and
finally, Dwight Schrute also joined in 2001 giving him 14 points as well. They decide on a
quota of 58 points. The weighted voting system can be represented by: [58: 30, 24, 14, 14]

*(handwritten annotations:)* 52 *(over "58" first occurrence)*     52 *(over "58" second occurrence)*

$$24 + 14 = 38$$

$$24 + 14 + 14 = 38 + 14 = 52$$

So the last 3 players together don't form a winning coalition,
which means any winning coalition **needs to include** $P_1$.

Coalitions:

No individual coalitions.

$\{P_1, P_2\} = 30 + 24 = 54 > 52$

$\{P_1, P_3\} = 30 + 14 = 44$  ✗

$\{P_1, P_4\} = 30 + 14 = 44$  ✗

$\{P_1, P_2, P_3\} = 30 + 24 + 14 = 68$ ✓

$\{P_1, P_2, P_4\} = 30 + 24 + 14 = 68$ ✓

$\{P_1, P_3, P_4\} = 30 + 14 + 14 = 58 \geqslant 52$ ✓
   $30 + 14 < 52$

$\{P_2, P_3, P_4\} = 28 + 14 + 14 = 56 > 52$

$\{P_1, P_2, P_3, P_4\}$  None are critical!

| | # critical | Power |
|---|---|---|
| $P_1$: | 4 | $P_1$: $4/12 = 1/3$ |
| $P_2$: | 4 | $P_2$: $4/12 = 1/3$ |
| $P_3$: | 2 | $P_3$: $2/12 = 1/6$ |
| $P_4$: | 2 | $P_4$: $2/12 = 1/6$ |

total = 12

Also seems unfair: even though
Stanley worked longer than
Creed, they have the same
amount of power!
   However the two folks that
started in 2001 have the same
amount of power, which is good.