

## Worksheet 2: More Voting Theory (IRV / RCV)

Group Names: Solutions

1. A class is voting on what kind of ice cream to have. The choices are strawberry (S), chocolate (C), and vanilla (V). The students in the class ranked their ice cream choices and the following preference table was constructed.

# votes	2	5	1	2	2
1st choice	S	V	S	C	C
2nd choice	V	C	C	S	V
3rd choice	C	S	V	V	S

Find the winner under the Instant Runoff Voting (Ranked Choice Voting!) method, by answering the following:

- (a) Which flavor gets eliminated in round 1? Strawberry

1<sup>st</sup> choice:

total S =  $2 + 1 = 3$     total C =  $2 + 2 = 4$   
total V = 5

S has the smallest # of 1<sup>st</sup> choice votes.

- (b) Construct the new preference table after the first elimination round.

# votes	2	5	1	2	2
1st choice	<del>S</del>	V	<del>S</del>	C	C
2nd choice	V	C	C	<del>S</del>	V
3rd choice	C	<del>S</del>	V	V	<del>S</del>

1<sup>st</sup> choice    2    5    1    2    2  
V    V    C    C    C  
2<sup>nd</sup> choice    C    C    V    V    V

	7	5
1 <sup>st</sup>	V	C
2 <sup>nd</sup>	C	V

- (c) Who is the IRV (RCV) winner? Vanilla

- (d) Do you think the IRV winner accurately represents the class's preference for ice cream? Explain your answer in a sentence or two.

Yes, because vanilla was preferred by a majority (7/12) of the students, even though it wasn't everyone's first choice.

2. Consider the following preference schedule for an election, with choices Abbot (a), Bingham (b), Chowdhury (c), and Dennison (d).

	5	6	12	3	6	3	3	3
1st choice	d	d	c	c	b	b	d	a
2nd choice	a	a	a	a	c	a	c	c
3rd choice	b	c	d	b	d	d	b	d
4th choice	c	b	b	d	a	c	a	b

$$5 + 6 + 12 + 3 + 6 + 3 + 3 + 3 = 41$$

$$\frac{41}{2} = 20.5$$

(a) How many people voted? 41 What is needed for a majority? 21

(b) How many possible rounds of IRV/RCV might this election require? How do you know?

Two rounds, to eliminate the 3<sup>rd</sup> & 4<sup>th</sup> choices, if no one gets a majority sooner.

(c) Who is the plurality winner? Chowdhury Do they have a majority? No!

$$a = 3 \quad c = 12 + 3 = 15$$

$$b = 6 + 3 = 9 \quad d = 5 + 6 + 3 = 14$$

they may have  $15 < 20.5$ .

(d) Determine the IRV/RCV winner, if one exists. Show what choices you made at each step, along with the necessary preference schedules.

At round 2, a is eliminated.

	5	6	12	3	6	3	3	3
1st choice	d	d	c	c	b	b	d	<del>a</del>
2nd choice	<del>a</del>	<del>a</del>	<del>a</del>	<del>a</del>	c	<del>a</del>	c	c
3rd choice	b	c	d	b	d	d	b	d
4th choice	c	b	b	d	<del>a</del>	c	<del>a</del>	b

	5	6	12	3	6	3	3	3
1 <sup>st</sup>	d	d	c	c	b	b	d	c
2 <sup>nd</sup>	b	c	d	b	c	d	c	d
3 <sup>rd</sup>	c	b	b	d	d	c	b	b

	5	9	15	3	6	3
1 <sup>st</sup>	d	d	c	c	b	b
2 <sup>nd</sup>	b	c	d	b	c	d
3 <sup>rd</sup>	c	b	b	d	a	c

total c = 18  
total b = 9  
total d = 14

No one has a majority  
b has the fewest votes.

Round 3. b is eliminated

	5	9	15	3	6	3
1 <sup>st</sup>	d	d	c	c	<del>b</del>	<del>b</del>
2 <sup>nd</sup>	<del>b</del>	c	d	<del>b</del>	c	d
3 <sup>rd</sup>	c	<del>b</del>	<del>b</del>	d	d	c

→

	5	9	15	3	6	3
1 <sup>st</sup>	d	d	c	c	c	d
2 <sup>nd</sup>	c	c	d	d	d	c

→

	5+9+3	15+3+6
	17	24
1 <sup>st</sup>	d	c
2 <sup>nd</sup>	c	d

Final round: c gets 24  
d gets 17  
c wins!

Who won the election? Chowdhury

Extra question: construct a 3-choice preference schedule where the IRV winner is different from the plurality winner.

In actual elections, this outcome is fairly rare but not impossible. (It's pretty hard to do this) We need the 2<sup>nd</sup> choice winner to be preferred by most of the 3<sup>rd</sup> choice folks, for sure. And we need there to not be a majority winner in round 1. Let's start by having an almost equal distribution. Here, b & c are splitting the votes of a lot of folks.

Round 1

	12	11	10
1 <sup>st</sup>	a	c	b
2 <sup>nd</sup>	b	b	c
3 <sup>rd</sup>	c	a	a

← total = 33 votes

← Candidate a wins the plurality vote with  $12/33 = 36\%$

← Candidate b is eliminated.



Round 2

	12	11	10
1 <sup>st</sup>	a	c	c
2 <sup>nd</sup>	c	a	a



	12	21
1 <sup>st</sup>	a	c
2 <sup>nd</sup>	c	a

So in round 2, candidate c wins decisively,  $21/33 = 64\%$