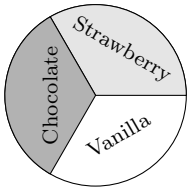
Worksheet 7 (Fair Division 2): The Lone Divider Method

Group Names: Solutions

Tom, Fred, and Janet are dividing a super-fancy gourmet cake worth $36 that is equal parts strawberry, vanilla and chocolate.



1. How much value is a fair share of the cake? $12 $36/3 = $12

2. Tom divides the cake into three pieces (not necessarily the division shown above!) that he values equally. Janet and Fred value the pieces according to the following table:

	piece 1	piece 2	piece 3
Tom	$12	$12	$12
Janet	$7	$18	$11
Fred	$15	$18	$3

   (a) Which pieces represent a fair share for Janet? piece 2

   (b) Which pieces represent a fair share for Fred? piece 2 & 3

   (c) Is it possible to distribute the pieces of cake to the three people so that everyone gets a piece that is a fair share for them? If so, explain how to do so; if not, explain what happens next.

   Yes: Janet gets piece 2, Fred gets Piece 3, and Tom gets piece 1.

3. It turns out that Janet and Fred changed their mind on how they value the pieces of cake that Tom gave. Their new values are given in the following table:

	piece 1	piece 2	piece 3
Tom	$12	$12	$12
Janet	$6	$20	$10
Fred	$7	$18	$11

   (a) Which pieces represent a fair share for Janet? piece 2

   (b) Which pieces represent a fair share for Fred? piece 2

   (c) Is it possible to distribute the pieces of cake to the three people so that everyone gets a piece that is a fair share for them? If so, explain how to do so; if not, explain what happens next.

   No! Give Tom piece 1 (arbitrarily between 1 & 3) and then Janet & Fred need to use divider-chooser.

4. In a final cake scenario, suppose that the people valued the cake as follows:

- Tom likes all three flavors equally;
- Janet likes strawberry twice as much as vanilla. She likes chocolate 6 times as much as vanilla. ~~Janet likes chocolate 10 times as much as vanilla and strawberry, which she likes equally~~;
- Fred also likes vanilla and strawberry equally, but he likes chocolate ~~3~~ 4 times as much as vanilla and strawberry.

(a) If Tom portioned the cake into three pieces where each piece was a single flavor, determine the valuations that Janet and Fred would assign to the pieces of cake.

You can guess these values. But by algebra:

Janet:
$v + 2v + 6v = 36$
$9v = 36$
$v = 4$

	vanilla	chocolate	strawberry
Tom	\$12	\$12	\$12
Janet	\$ 4	\$ 24	\$ 8
Fred	\$ 6	\$ 24	\$ 6

Fred:
$4v + v + v = 36$
$6v = 36$
$v = 6$

(b) Which pieces represent a fair share for Janet? only chocolate

(c) Which pieces represent a fair share for Fred? only chocolate

(d) Explain why it is not possible to distribute Tom's pieces of cake so that everyone gets a fair share.

Janet & Fred both only value chocolate as their fair share, among this cake partition.

(e) Choose a piece of cake to assign to Tom, and explain why you chose that piece.

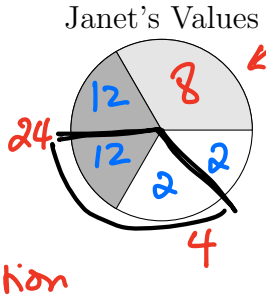
Tom gets either vanilla or strawberry. I choose strawberry.

(f) Now, use Divider-Chooser to determine the rest of the division of the cake. Suppose that you flipped a coin, and Janet was chosen to be the divider.

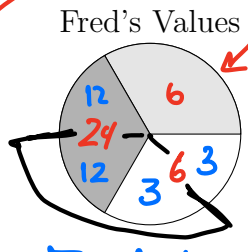
i. Label Janet's values on the cake and draw a partition of the cake on Janet's side that Janet might make as the divider.

ii. Label Fred's values, determine which part of the cake he will choose, and determine the value of that part of the cake to him.

Janet needs at least \$12 of value. But she has \$28 to work with! She needs to divide into 2 equal shares for her. In this partition they are both worth \$14 to her.



Not available for distribution!



Fred chooses either one of Janet's pieces because they have the same value for him. He values his piece at \$15.