Voting Theory

You should know

Terminology: majority, plurality, Cordorcet Winner

Voting Methods by Name: Plurality Method, Instant Run-off Voting (IRV), Borda Count, Copeland's Method

Sample Voting Theory Problem

A class of middle schoolers are trying to decide what would be the worst ingredient to add to a cake their Principal has to eat if they win the Science Bowl. They narrow their options down to three – pickled onions (PO), stinky cheese (SC), or anchovies (A). The class is polled and the resulting preference schedule is below.

	22	5	23	2	11	20
1st choice	PO	PO	SC	SC	A	A
2nd choice	SC	A	PO	A	PO	SC
3rd choice	A	SC	A	PO	SC	PO

1. How many students voted?
2. How many votes does a candidate need to win in order to win a majority? Show the calculation that gives your answer.
3. How many votes does a candidate need to win in order to win a plurality? Show the calculation that gives your answer.
4. Find the winner using the plurality method. Show your work.
5. Find the winner using Instant Runoff Voting. Show your work.

	22	5	23	2	11	20
1st choice	PO	PO	SC	SC	A	A
2nd choice	SC	A	PO	A	PO	SC
3rd choice	A	SC	A	PO	SC	PO

6. Find the winner using Borda Count. Show your work.

7. Find the winner using Copeland's Method. Show your work.

8. Is any candidate a Condorcet Winner? Explain your answer.

Weighted Voting

Terminology: quota, weight, winning coalition, critical player in a winning coalition, dictator, veto power, dummy, Banzhaf Power Index or Banzhaf Power Distribution.

Sample Weighted Voting Problems

1. For each weighted voting system below, determine if there are any dictators, anyone with veto power, or any dummies.

   (a) $[16 : 16, 11, 3, 1]$

   (b) $[51 : 40, 30, 20, 10]$

   (c) $[31 : 10, 9, 8, 7, 6]$

2. Consider the weighted voting system $[q : 21, 20, 7, 5]$ where the players can pass a motion with a **majority**.

   (a) What is q in this case?

   (b) List all winning coalitions.

   (c) In each of the winning coalitions above, indicate who is a critical player.

   (d) Calculate the Banzhaf Power Index for each player.

   (e) Based on your calculations above, are there any dummies? Explain.

Fair Division

Methods by Name: Divider-Chooser, Lone Divider, the Method of Sealed Bids.

Sample Fair Divison Problems

1. For each of the three methods listed above, what is the **context** in which they are appropriate?

2. (a) James, Gordon, Julie, Alexei, and Latrice divide their pile of Halloween candy worth a total of $20. Determine who was the divider and determine how the lone divider method would proceed.

	Pile 1	Pile 2	Pile 3	Pile 4	Pile 5
James	$0	$5	$10	$5	$0
Gordon	$2	$4	$6	$2	$6
Julie	$4	$4	$4	$4	$4
Alexei	$2	$4	$12	$2	$0
Latrice	$2	$10	$7	$1	$0

(b) Suppose **Gordon** changes his evaluation of the piles. Now determine how the lone divider method would proceed.

	Pile 1	Pile 2	Pile 3	Pile 4	Pile 5
James	$0	$5	$10	$5	$0
Gordon	$2	**$7**	$6	$2	**$3**
Julie	$4	$4	$4	$4	$4
Alexei	$2	$4	$12	$2	$0
Latrice	$2	$10	$7	$1	$0

3. The Cookie Monster and Elmo are splitting a giant cookie that is half chocolate-chip and half oatmeal worth $6. The Cookie Monster likes chocolate chip twice as much as oatmeal. Elmo likes oatmeal three times as much as chocolate chip. Suppose someone splits the cookie with all chocolate chip on one side and all oatmeal on the other. Attachdollar values to each share for each person.

	share 1: chocolate chip	share 2: oatmeal
Cookie Monster		
Elmo		

4. Harry Potter (P), Hermione Granger (G), Luna Lovegood (L), and Ronald Weasley (W) are dividing up some loot they found in the dungeons. The loot consists of a velvet cloak, a gold chalice, and a self-cleaning cauldron. They decide to divide the loot using the sealed bid method. The table below shows how many gold coins each person bid for each item.

	cloak	chalice	cauldron
Potter	60 gold coins	60 gold coins	0 gold coins
Granger	50 gold coins	10 gold coins	80 gold coins
Lovegood	100 gold coins	0 gold coins	100 gold coins
Weasley	30 gold coins	20 gold coins	30 gold coins

   (a) Determine each person's fair share.

   (b) Determine which person gets each item.

   (c) Determine how many gold coins each of them owes to the holding pile or receives from the holding pile.

(d) Determine the surplus. (Fractional gold coins are fine.)

(e) Determine the final allotment.