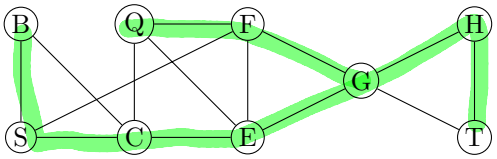
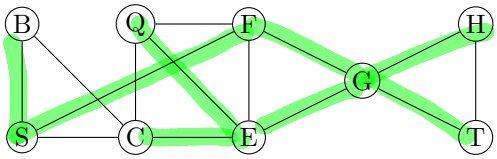
Worksheet 10 (Graph Theory 2): Minimum Cost Spanning Tree

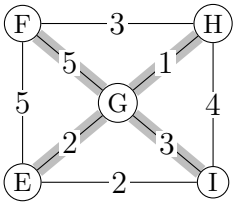
1. Find two different spanning trees in the graph below. Draw one on each copy of the graph.



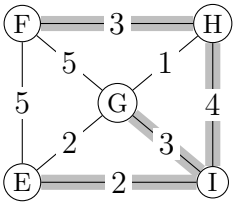


2. Here are four different subgraphs in a weighted graph. Graphs 1 and 2 are spanning trees.

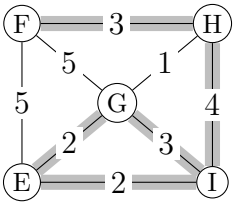
Graph 1



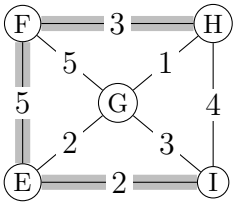
Graph 2



Graph 3



Graph 4



(a) What is the total cost of the spanning tree in Graph 1? 1+2+3+5=11

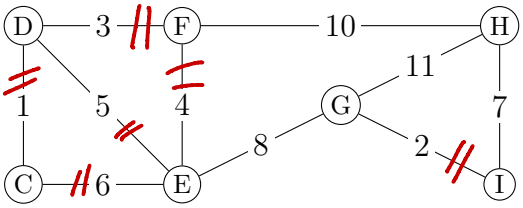
(b) What is the total cost of the spanning tree in Graph 2? 3+4+3+2=12

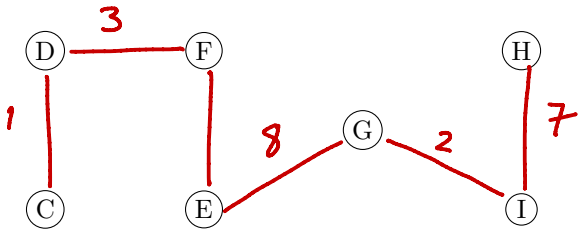
(c) Which spanning tree has smaller total cost? Graph 1

(d) Why is the subgraph in Graph 3 not a spanning tree? circuit : GEIG

(e) Why is the subgraph in Graph 4 not a spanning tree? not spanning

3. Use Kruskal's Algorithm to find a minimum cost spanning tree in the following graph. Construct the tree in the second graph. Keep track of the steps of the algorithm in the table below.





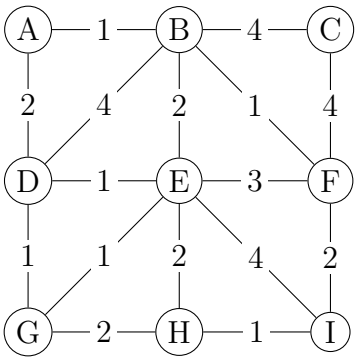
Used?	edges	weights
✓	CD	1
✓	GI	2
✓	DF	3
✓	EF	4
No	DE	5
No	CE	6
✓	HI	7
✓	EG	8

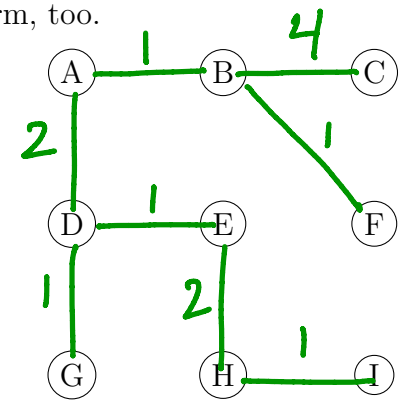
stop.

What is the total cost of the spanning tree you found? 1+2+3+4+7+8=25

4. Use Kruskal's Algorithm to determine a minimum cost spanning tree in the following graph.

   Break ties by choosing the edge that comes earlier in the alphabet. For convenience, the edges and weights are provided in tabular form, too.





Sorted edges	weight	used
AB	1	✓
BF	1	✓
DE	1	✓
DG	1	✓
EG	1	NO
HI	1	✓
AD	2	✓
BE	2	NO
EH	2	✓
FI	2	NO
GH	2	NO
EF	3	NO
BC	4	✓
BD	4	
CF	4	
EI	4	

Stop

What is the total cost of the spanning tree you found? $1+1+1+1+1+2+2+4 = 13$

5. Why are there instructions about how to break ties? Is this important? What are the consequences?

- To be an algorithm that a computer can execute requires instructions that are specific and predetermined.
- For large graphs you need a computer to help.
- On small graphs, it ensures we all get the same answer!

6. (Bonus) How many edges could a graph on 9 vertices have? 36. Already alot to check.