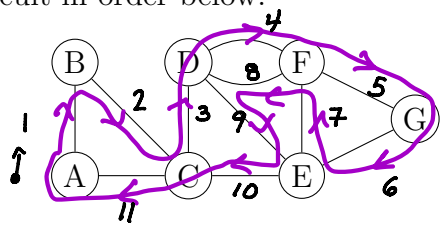
Worksheet 12 (Graph Theory 4): Euler Paths and Circuits

Group Names: Solutions

1. For each of the following graphs, first **highlight all the vertices of odd degree.**

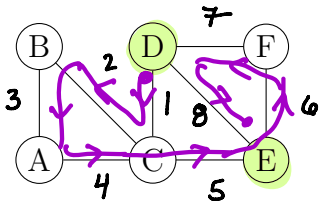
   (a) Find an Euler circuit in this graph.

   Draw next to the edges of the graph so it is clear what the circuit/path should be, and list the vertices of the circuit in order below.



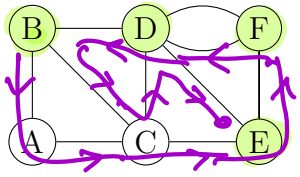
   Circuit: A B C D F G E F D E C A

   (b) Find an Euler path in this graph.



   Path: D C B A C E F D E

   (c) Try to find an Euler circuit or path in this graph.



   What happens when you try to find an Euler circuit or path in this graph?

   I can't get to some edges without retracing some other edges!

2. Make a **conjecture** (this is a fancy math word for "guess you think is true"): thinking about the degrees of vertices,

   When does a graph have an Euler circuit?

   When all vertices have even degree

   When does a graph have an Euler path but no circuit?

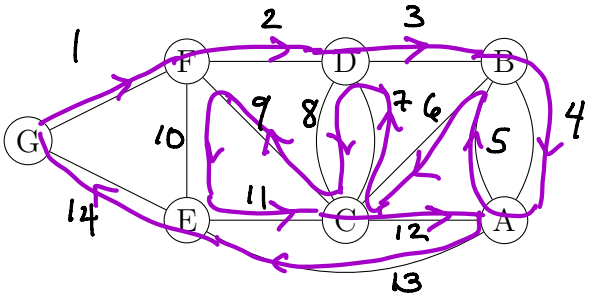
   When it has exactly 2 vertices of odd degree

   When does a graph have no Euler path and no Euler circuit?

   When it has more than 2 vertices of odd degree

3. Using your **conjecture**, do you think this graph has an Euler circuit or path? Circuit

   If there is an Euler circuit or path, draw it next to the edges of the graph so it is clear what the circuit should be, and list the vertices of the circuit in order.

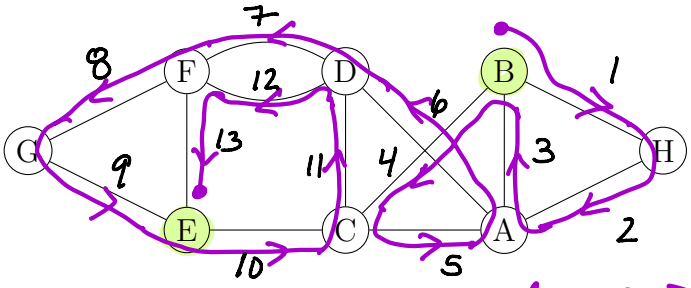


circuit/path: G F D B A B C D C F E C A E G

4. (a) This graph has two vertices of odd degree. What are they? E and B

   (b) Using your **conjecture**, do you think this graph has an Euler circuit or path? path

   If there is an Euler circuit or path, draw it next to the edges of the graph so it is clear what the circuit/path should be, and list the vertices of the circuit/path in order.



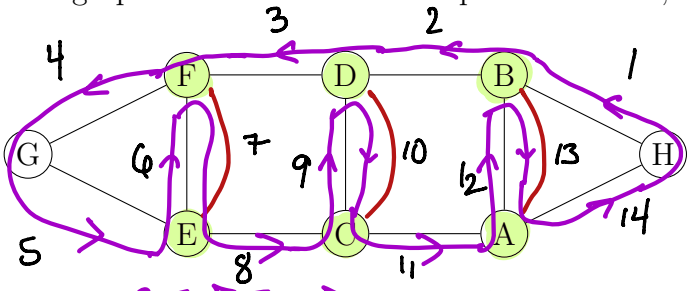
circuit/path: B H A B C A D F G E C D F E

5. (a) This graph has more than two vertices of odd degree.
   List the odd-degree vertices A B F E D C

   (b) **Double** some of the edges (that is, add a new edge next to an edge that's already there) so that every vertex is even degree. Which edges did you add?
   EF CD AB

   (c) What is the smallest number of edges you can add?
   3

   (d) Using your additional edges, find an Euler circuit. Draw it next to the edges of the graph so it is clear what the path should be, and list the vertices of the circuit in order.



circuit: G E F E C D C A B A H B D F G