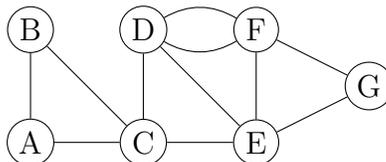


## Worksheet 12 (Graph Theory 4): Euler Paths and Circuits

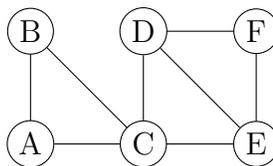
1. For each of the following graphs, first **highlight all the vertices of odd degree**. Indicate an Euler circuit/path on the graph by numbering the edges and adding a little arrow in the direction of travel.

- (a) Find an Euler circuit in this graph. Then list the vertices of the circuit in order below.



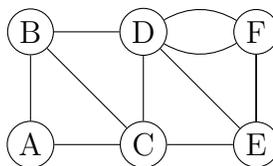
Circuit: \_\_\_\_\_

- (b) Find an Euler path in this graph.



Path: \_\_\_\_\_

- (c) Try to find an Euler circuit or path in this graph.



What happens when you try to find an Euler circuit or path in this graph?

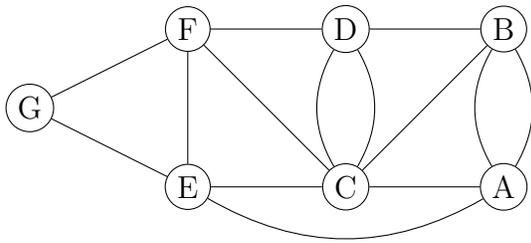
2. Make a **conjecture** (this is a fancy math word for “guess you think is true”): thinking about the degrees of vertices,

When does a graph have an Euler circuit?

When does a graph have an Euler path but no circuit?

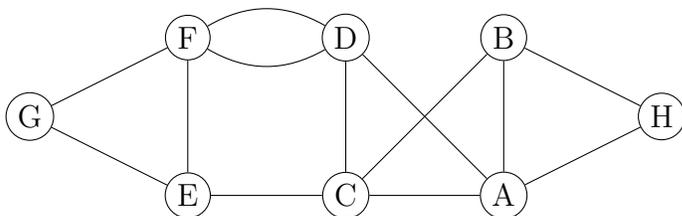
When does a graph have no Euler path and no Euler circuit?

3. Using your **conjecture**, do you think this graph has an Euler circuit or path? \_\_\_\_\_  
 If there is an Euler circuit or path, draw it next to the edges of the graph so it is clear what the circuit should be, and list the vertices of the circuit in order.



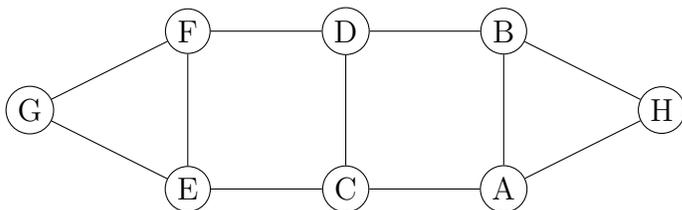
circuit/path: \_\_\_\_\_

4. (a) This graph has two vertices of odd degree. What are they? \_\_\_\_\_  
 (b) Using your **conjecture**, do you think this graph has an Euler circuit or path? \_\_\_\_\_  
 If there is an Euler circuit or path, draw it next to the edges of the graph so it is clear what the circuit/path should be, and list the vertices of the circuit/path in order.



circuit/path: \_\_\_\_\_

5. (a) This graph has more than two vertices of odd degree.  
 List the odd-degree vertices \_\_\_\_\_  
 (b) **Double** some of the edges (that is, add a new edge next to an edge that's already there) so that every vertex is even degree. Which edges did you add?  
 \_\_\_\_\_  
 (c) What is the smallest number of edges you can add?  
 \_\_\_\_\_  
 (d) Using your additional edges, find an Euler circuit. Draw it next to the edges of the graph so it is clear what the path should be, and list the vertices of the circuit in order.



circuit: \_\_\_\_\_