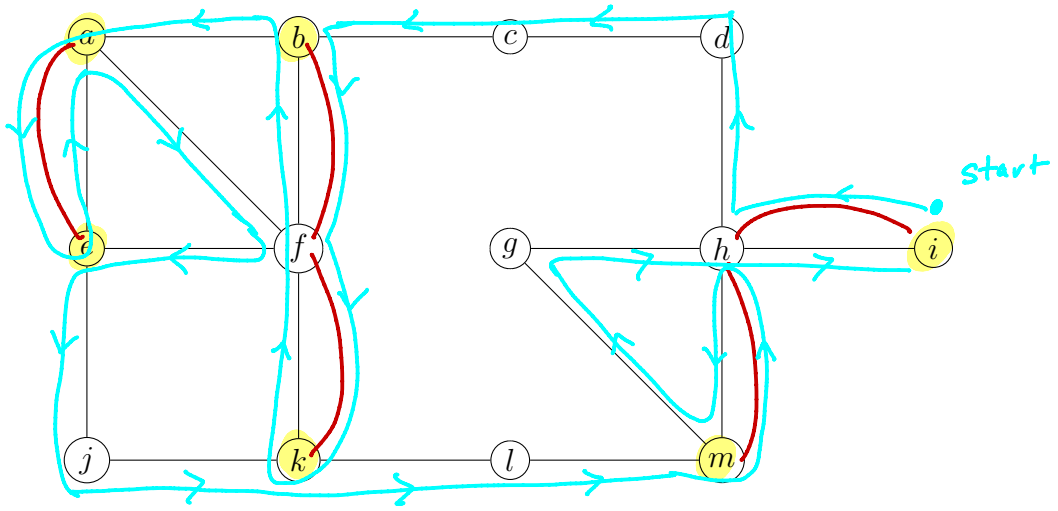
Worksheet 13 (Graph Theory 5): Eulerization

1. Consider the following graph.

   (a) Which are the vertices of odd degree? a, b, e, k, i, m

   (b) **Eulerize this graph:** find the smallest number of edges you can duplicate so that you can construct an **Euler circuit**, and add them to the graph.

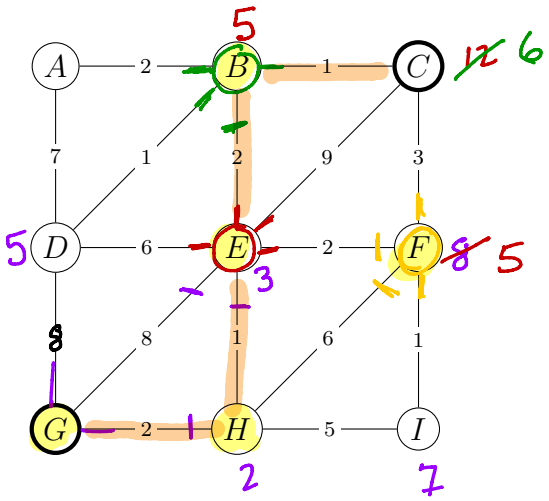
   (c) Draw the circuit on the graph (offset slightly so you can see it) so that you can follow it without lifting your pencil from the circuit.



   (d) Give this graph a context: What might this graph represent? Why might you need an Euler circuit?

- Vertices are intersections of streets
- Edges are roads between intersections.
- Euler circuit is an efficient way to plow all the streets.

2. Consider the following weighted graph.



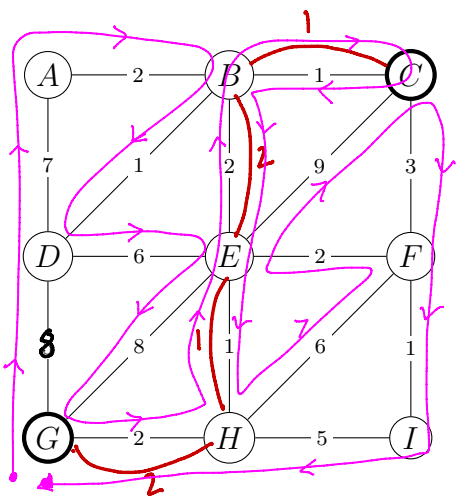
(a) There are two vertices of odd degree in this graph, c and g. Use Dijkstra's algorithm to find a path of minimum (weighted) distance from c to g. Break ties by using alphabetical order. List the vertices in order you explored them to the right.

vertex	A	B	C	D	E	F	G	H	I
current/visited		✓	done		✓		✓	✓	✓
tentative distance	7	5 ✓	6	B	3 ✓	5	0 ✓	2 ✓	6 ✓
preceding vertex	B	E	B	G	H	E	-	G	F

Order Explored: G, H, E, B, F, C

What is your shortest-weighted-distance path? C B E H G

(b) On the copy of the graph below, **duplicate your minimum distance path** (including the weights) to eulerize the graph. Then find an **Euler circuit** in the graph, which will be of minimum total weight.



ANS: We simply need to sum all the weights.

15
21
6
13
11
4
70

Note: We do not need the circuit to know its weight is 70.