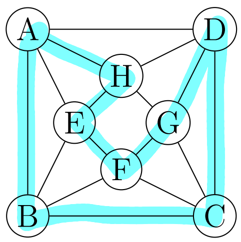
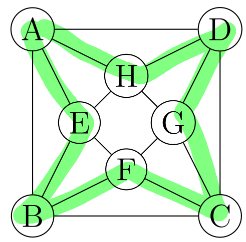
Worksheet 14 (Graph Theory 6): Hamiltonian Circuits

1. Draw two different Hamiltonian circuit in the graph below. Below each graph, list the vertices of your circuit in order.

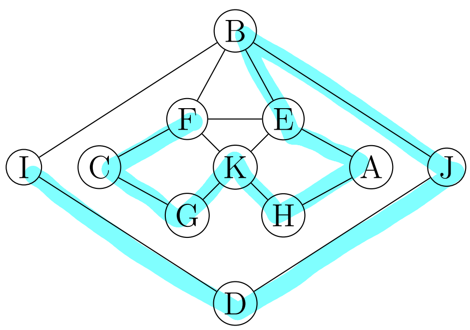


ABCDGFEHA



AHDGCFBEA

2. Answer questions about the graph sketched below.



(a) Draw a Hamiltonian path starting at vertex I. List the vertices of your circuit in order.

IDJBEAHKGCF

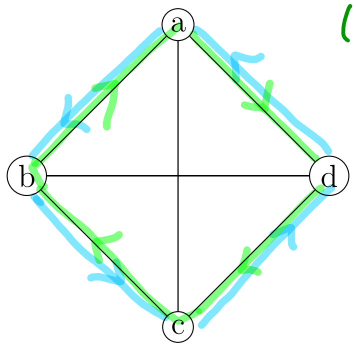
(b) Draw a Hamiltonian path starting at vertex B or explain why this is not possible.

It's not possible. B is the only vertex connecting I J and D to the remaining vertices.

(c) Find a Hamiltonian *circuit* or explain why this is not possible.

It's not possible. For the same reason as in b

3. List *every* possible Hamiltonian circuit in the graph below. Give a numerical justification that you have all of them.



(There are many ways to answer this.) I will assume it always starts at a. So the number of choices is: a $\underset{3}{\_}\ \underset{2}{\_}\ \underset{1}{\_}$ or $3! = 3\cdot2\cdot1 = 6$.

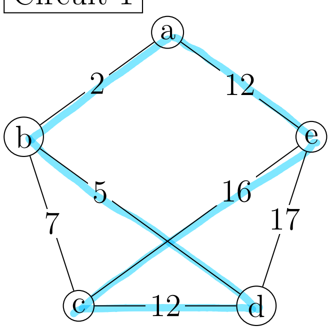
1 abcda
abdca
acbda
acdba

adbca
adcba 2

Someone else might consider 1 and 2 to be the same, since one is the reverse of the other.

4. The following graph has two different Hamiltonian circuits. Highlight one on each copy of the graph and compute the total weight of the circuit.

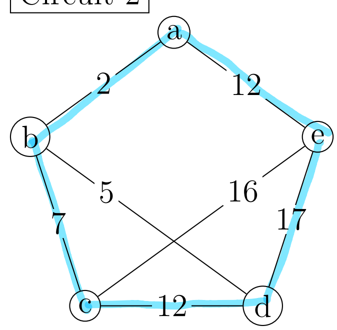
Circuit 1



$$\begin{array}{r} 14 \\ 21 \\ 12 \\ \hline 47 \end{array}$$

Weight: 47

Circuit 2



2:
$$\begin{array}{r} 14 \\ 19 \\ 17 \\ \hline 50 \end{array}$$

Weight: 50

Which Hamiltonian circuit has the smallest weight? Circuit 1

5. Recall that Kruskal's Algorithm found a minimum weight spanning tree by selecting the cheapest edges that don't form a circuit. Do you think such an algorithm can be modified to find a minimum weight Hamiltonian circuit? What modifications would be needed? What might be some challenges?

- Instead of avoiding a circuit, we have to avoid closing a circuit too soon
- Once we are close to a circuit, we don't have any choice about what edges to use.