Worksheet 15 (Graph Theory 7): Hamiltonian Circuits & (Repeated) Nearest Neighbor

1. Below is a table of great-circle distances between the six most populous cities in Africa.

Table 1: Great-circle distances (km) between the 6 most populous cities in Africa (2025). Cities: Cairo (Egypt), Lagos (Nigeria), Kinshasa (DRC), Luanda (Angola), Dar es Salaam (Tanzania), Johannesburg (South Africa). A dash (—) indicates the diagonal. Distances are symmetric.

	Cairo	**Lagos**	**Kinshasa**	**Luanda**	**Dar es Salaam**	**Johannesburg**
Cairo	—	3,913	4,179	4,732	4,184	6,264
Lagos	3,913	—	1,798	2,027	4,242	4,508
Kinshasa	4,179	1,798	—	554	2,651	2,781
Luanda	4,732	2,027	554	—	2,870	2,484
Dar es Salaam	4,184	4,242	2,651	2,870	—	2,461
Johannesburg	6,264	4,508	2,781	2,484	2,461	—

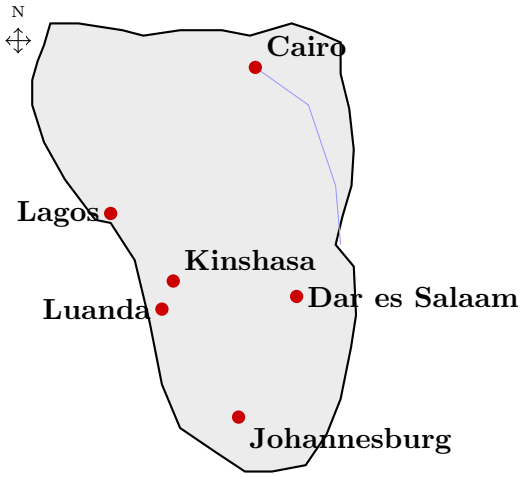
(a) How many different Hamiltonian circuits are possible?

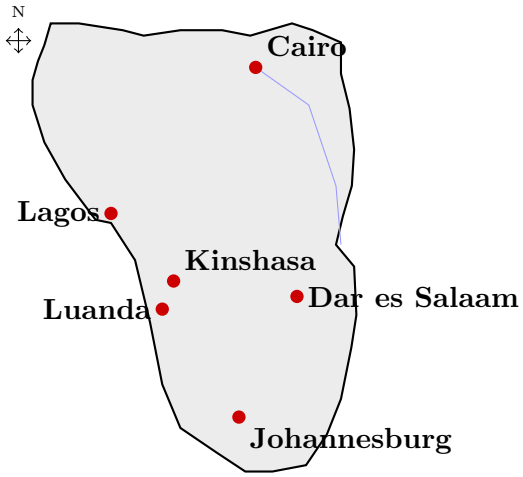
(b) Use the Nearest Neighbor algorithm starting at the specified vertex to find a Hamiltonian circuit and determine its weight.

i. Start at Johannesburg.

ii. Start at Cairo.

2. Below are two rough sketches Africa with the cities from the previous problem. Sketch your Hamiltonian circuits from the previous problem. What do you observe? Do you think you have found the shortest tour?





3. Add weights to the edges of a complete graph on four vertices such that the Nearest Neighbor Algorithm starting at vertex A selects a Hamiltonian circuit that has very high weight.