Worksheet 15 (Graph Theory 7): Hamiltonian Circuits & (Repeated) Nearest Neighbor

1. Below is a table of great-circle distances between the six most populous cities in Africa.

Table 1: Great-circle distances (km) between the 6 most populous cities in Africa (2025). Cities: Cairo (Egypt), Lagos (Nigeria), Kinshasa (DRC), Luanda (Angola), Dar es Salaam (Tanzania), Johannesburg (South Africa). A dash (—) indicates the diagonal. Distances are symmetric.

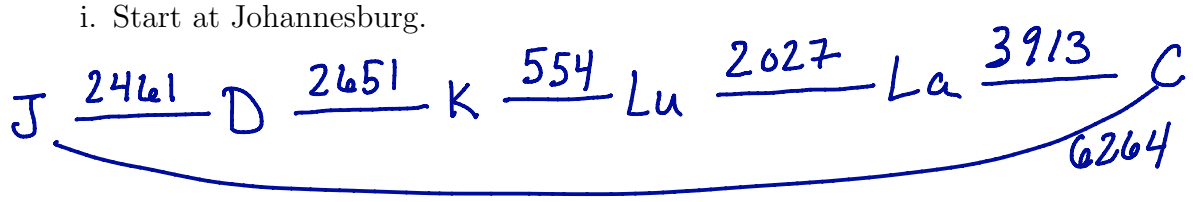
	Cairo	Lagos	Kinshasa	Luanda	Dar es Salaam	Johannesburg
C **Cairo**	—	3,913	4,179	4,732	4,184	6,264
La **Lagos**	3,913	—	1,798	2,027	4,242	4,508
K **Kinshasa**	4,179	1,798	—	554	2,651	2,781
Lu **Luanda**	4,732	2,027	554	—	2,870	2,484
D **Dar es Salaam**	4,184	4,242	2,651	2,870	—	2,461
J **Johannesburg**	6,264	4,508	2,781	2,484	2,461	—

(a) How many different Hamiltonian circuits are possible?

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$. That's a lot!

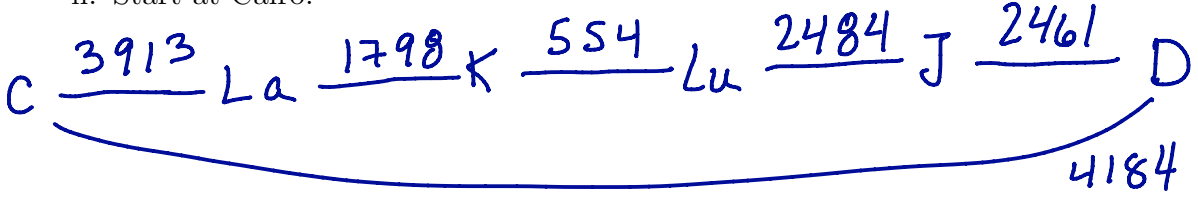
(b) Use the Nearest Neighbor algorithm starting at the specified vertex to find a Hamiltonian circuit and determine its weight.

i. Start at Johannesburg.



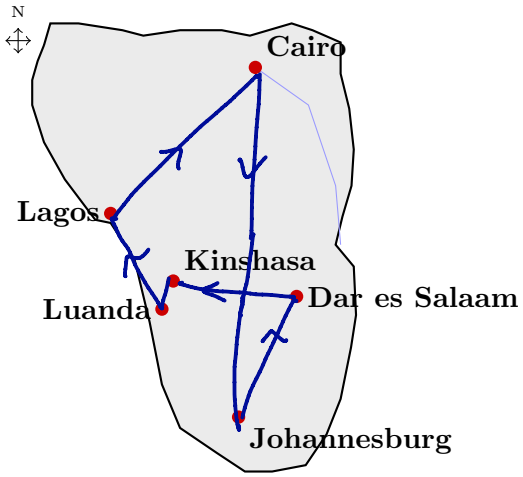
total: 15,845 km

ii. Start at Cairo.

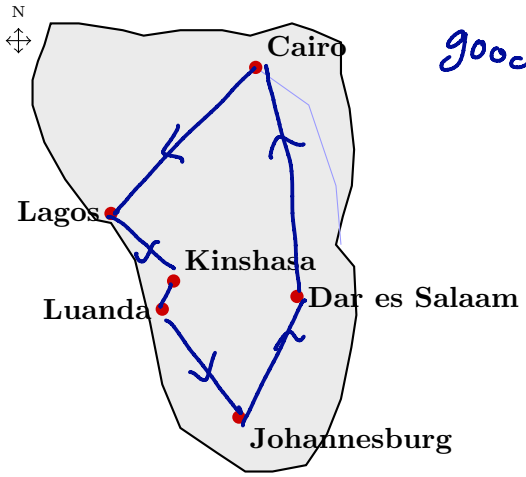


total: 15,394 km

2. Below are two rough sketches Africa with the cities from the previous problem. Sketch your Hamiltonian circuits from the previous problem. What do you observe? Do you think you have found the shortest tour?



This route looks good.



3. Add weights to the edges of a complete graph on four vertices such that the Nearest Neighbor Algorithm selects a Hamiltonian circuit that has very high weight.