

Worksheet 5: Weighted Voting and Banzhaf Power Index

1. The weighted voting system, $[12 : 11, 8, 4]$, assigned voting weights to players P_1 , P_2 , and P_3 according to how many hours per week they worked at a co-owned business.
 - (a) Given this system, what is your *intuition* regarding the distribution of power among the players? Make a rough estimate for how you think the power is distributed. (This means assigning a percentage to each player such that the numbers sum to 100%.)
 - (b) List all possible winning coalitions. (Hint: There are 4.)
 - (c) Calculate the Banzhaf power distribution.
- (d) Are you surprised? Do you think this system is operating as the players intended?
- (e) Explain why the quota cannot be any smaller than 12 or any larger than 23.
- (f) Re-calculate the Banzhaf power distribution with the quota changed from 12 to 13.
- (g) What other changes to this voting system could shift the Banzhaf power distribution?

2. A condo community is voting to approve a \$10K loan to fix structural issues with the building. If passed, the homeowners are responsible for paying off the loan over the next five years which would result in an increase in the homeowners' monthly HOA fee. There are 2 homeowners (H_1 and H_2) and 2 board members (B_1 and B_2) on the committee that will vote on the loan proposal.

For this measure to pass both homeowners and at least one board member must vote yes.

(a) Construct a weighted voting system that represents this setup. You will need to identify which weights are associated with the homeowners (H_1 and H_2) and which are associated with the board members, (B_1 and B_2).

(You may need to try several options! See the next question to determine if your system works.)

(b) Show your system works, by verifying the following two conditions:

i. Every coalition of the form HHB or HHBB makes quota

ii. Every other coalition does NOT make quota