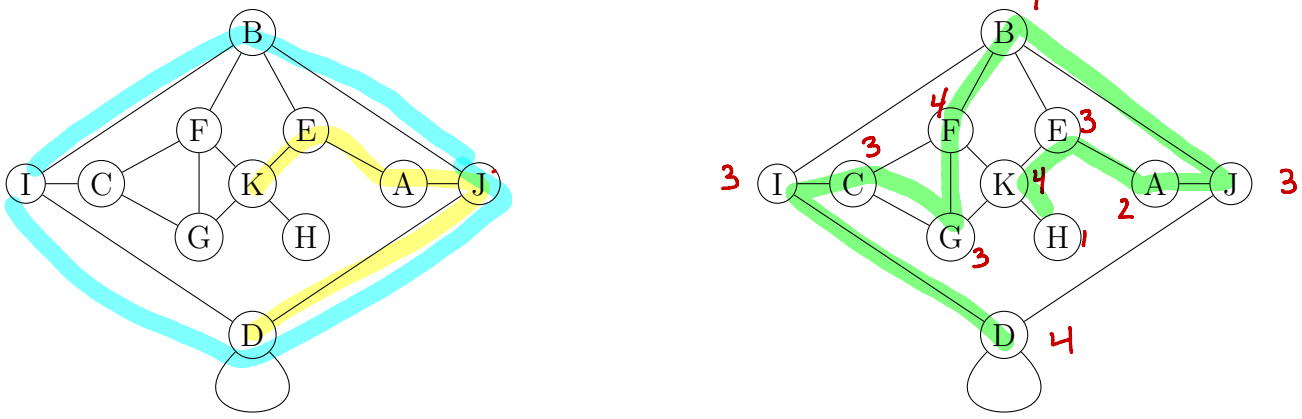
# Worksheet 9 (Graph Theory 1): Pieces of Graphs

1. Graph **Q** is shown below (twice). Answer the questions:




(a) How many vertices does graph **Q** have? 11

(b) How many edges does graph **Q** have? 17

(c) Degree of vertex $A$? 2

(d) Degree of vertex $H$? 1

(e) Degree of vertex $D$? (remember, loops count twice) 4

(f) Label each vertex on the right-hand copy of the graph with its degree.

(g) Which vertex/vertices has/have the largest degree? B, F, K, D (degree 4)

(h) Find a path from $K$ to $D$. Draw it on the (left-hand) graph. How many edges does your path have? _____

(i) What is the length of the shortest path from $I$ to $J$? 2 Write the path here: IBJ

(j) Find a circuit in the graph and highlight it on the graph. Write the circuit here: IBJDI

(k) Find a path that visits every vertex exactly once. Highlight it on the right-hand copy of the graph.

(l) Explain why you can't find a circuit that passes through every vertex of the graph.

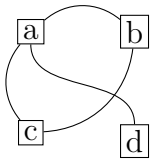
Degree of H is 1.

(m) Create a context for this graph. What might the vertices represent? What might the edges represent?

Vertices: people

edges: There is an edge between two people if they have ever taken a class together.

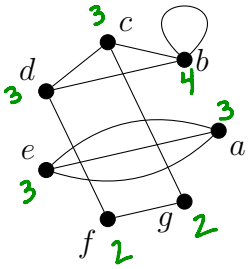
2. Here's a small graph, Graph **T**. To the right, represent it as a table (technically, as an "adjacency matrix"), by writing **1** in the spot in the row/column intersection if there's an edge between the row-vertex and column-vertex and 0 otherwise. For example, since there is an edge from vertex a to vertex d, we put a 1 in the first row, fourth column.

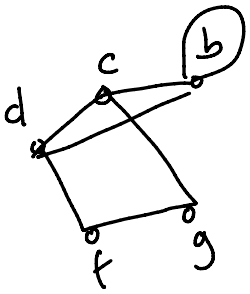



| | a | b | c | d |
|---|---|---|---|---|
| a | 0 | 1 | 1 | 1 |
| b | 1 | 0 | 1 | 0 |
| c | 1 | 1 | 0 | 0 |
| d | 1 | 0 | 0 | 0 |

Find a circuit in the graph. List the vertices in the circuit: ABCA

3. Here's another graph, Graph **R**.





(a) Explain why this graph is not connected.

There is no path from a to b

edge does NOT work here!

(b) A *connected component* is a piece of a graph that *is* connected. To the right of the graph, draw the two connected components of graph R separately, with no crossing edges. (You will need to change the position of the vertices and edges!)

(c) Label each vertex with its degree.

(d) How many edges does graph R have? 10

4. Using our previous graphs, fill in the following table:

| Graph | sum of the degrees | number of edges |
|---|---|---|
| Graph Q | 34 | 17 |
| Graph T | 8 | 4 |
| Graph R | 20 | 10 |

**Fill in the blank:** The sum of all the degrees of the vertices of a graph is double the number of edges of the graph.