Graph Theory

1. Define the terms below.

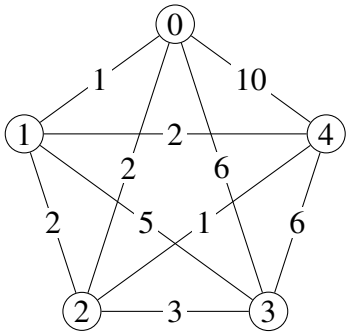
   (a) An Euler circuit is

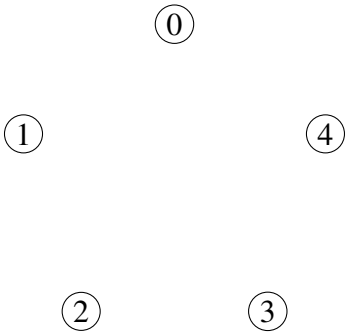
   (b) An Euler path is

   (c) A Hamiltonian circuit is

   (d) A spanning tree is

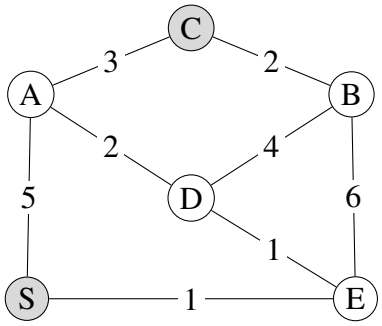
2. (a) Use Kruskal's Algorithm to find a minimum weight spanning tree in the graph below.





   (b) Give an example of a real-world problem which you would want to find a minimum weight spanning tree. (You would need to state what the vertices, edges, and weights represent and the significance of a minimum weight spanning tree in this context.)

3. (a) Use Dijkstra's Algorithm to find a shortest path from vertex *S* to vertex *C*.



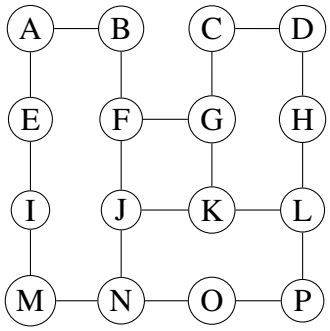
vertex	current/ visited	**tentative** minimum distance to *C*	preceding vertex
A			
B			
C			
D			
E			
S			

(b) State the shortest path from *S* to *C* and it's length/weight.

(c) Give an example of a real-world problem which you would want to find the minimum distance from *S*. (You would need to state what the vertices, edges, and weights represent and what is the significance of the shortest path in this context.)

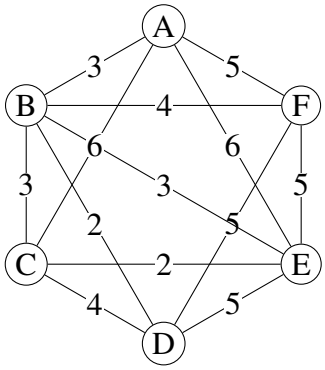
4. (a) What does it mean to Eulerize a graph? What is your goal?

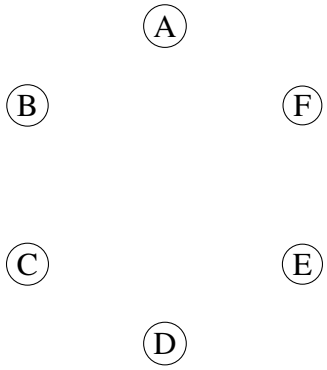
(b) Eulerize the graph below by adding the fewest number of edges and then find an Euler circuit in the resulting graph.



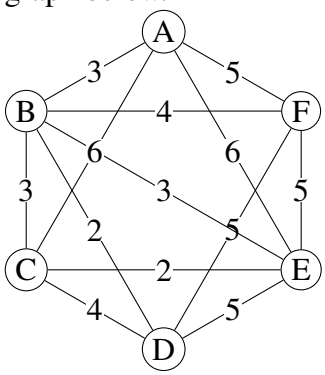
(c) Give an example of a real-world problem which you would want to find an Euler circuit. (You would need to state what the vertices, edges, and weights represent and what is the significance of the shortest path in this context.)

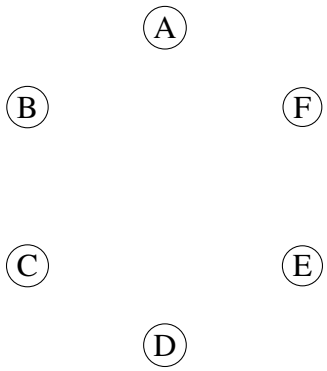
5. (a) Use the Nearest Neighbor Algorithm starting a vertex A, to find a Hamiltonian Circuit in the graph below and determine its weight.





(b) Use the Cheapest Link Algorithm to find a minimum weight Hamiltonian Circuit in the graph below.





(c) Give an example of a real-world problem which you would want to find a minimum weight Hamiltonian circuit. (You would need to state what the vertices, edges, and weights represent and what is the significance of the shortest path in this context.)

Scheduling

6. The table below contains the tasks to be completed for a project.

task	time (in minutes)	dependencies/ prerequisites
A	2	
B	9	
C	3	
D	7	A
E	1	C
F	6	A
G	4	E
H	8	D,F
I	5	G
J	1	B,H,I

   (a) To the right, above, create a digraph representing the project.

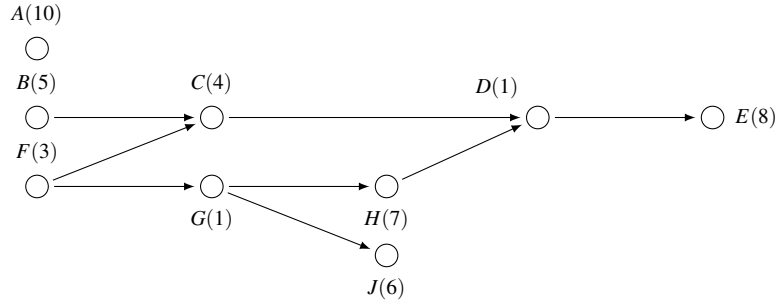
   (b) Find a critical path and its corresponding critical time. Explain the significance of the critical time.

   (c) Create a priority list using the decreasing time algorithm. (That is, create a priority list ordered by decreasing time.)

   (d) Use the Backflow algorithm to assign critical numbers to each vertex in the graph above.

   (e) Create a priority list using the critical time algorithm. (That is, create a priority list ordered by decreasing **critical** time.)

7. Use the diagraph below and the given priority list to construct a schedule using two processors.



Construct a schedule using the priority list

$$F, B, G, C, A, H, J, D, E$$

time	ready	done